THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XX SEPTEMBER 1904 NUMBER 2

ON THE OXYGEN ABSORPTION BANDS OF THE SOLAR SPECTRUM.

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Spectroscopists have long taken a special interest in those peculiar bands of the solar spectrum of which B is a typical and familiar example. Singly and collectively they have formed the subject of many investigations, especially since instruments of research began to be powerful enough to reveal, at least in some degree, their peculiar structure. The early researches of Egoroff,[1] Janssen,[2] and Thollon[3] had for their chief aim the discovery of the element to which these bands were to be attributed, with the result that they were shown to be due, at least in part, to the absorption of solar energy by the dry oxygen of the Earth's atmosphere. The later work of Cornu,[4] Janssen,[5] Dunér,[6] and others furnishes almost conclusive evidence that they are due solely to this cause. After the introduction of still more powerful and perfect spectroscopes, attention was drawn to the remarkable geometrical structure of each of these bands and to their striking similarity. From the latter point of view the most

[1] *C. R.*, **92**, 385, 1881; *ibid.*, **97**, 555, 1883.

[2] *Ibid.*, **107**, 1889; several communications.

[3] *Jour. de Phys.*, (2) **3**, 1884.

[4] *Ann. de Chim. et Phys.*, **7**, 5–102, 1886.

[5] *C. R.*, **107**, 672, 1889.

[6] *Ibid.*, **118**, 1894.

important studies have been those of Cornu,[1] Langley,[2] Piazzi Smyth,[3] Higgs,[4] and Deslandres.[5]

Langley and Smyth point out only the general resemblance as seen from their early photographs and drawings. Cornu attempted to establish certain more definite relations, but his wave-lengths were not measured very accurately and the relations to which he called attention were only roughly approximate. Higgs confines himself to the study of the relations of the lines in a single band, taking B as an example, and shows that if the positions of the lines in any series are laid off in wave-lengths along the X-axis and lines drawn through them parallel to Y, while on Y are laid off equal divisions and lines drawn parallel to X, then the intersections of the two systems of lines lie very closely on a parabola whose equation is of the form

$$\lambda = V + \frac{(n+c)^2}{p} ,$$

where n is any number of units from the vertex of the curve, and V, p, and c are constants. There are two parallel parabolas corresponding to the two series in each band, the vertices coinciding with the beginning of each series.

Deslandres points out a general similarity between A, B, and *a*, and certain bands discovered by Huggins, Liveing and Dewar, and himself in the ultra-violet spectrum of water-vapor. The water-vapor bands reproduce the oxygen bands on a larger scale, the ratio of the corresponding A groups, for example, being about 12 to 1. The chief differences are that the water-vapor groups as given by Liveing and Dewar[6] show no separation of head and tail, and, according to Deslandres,[7] are composed of two series instead of one. It will be seen later that the analogy is even closer than Deslandres supposed, as these two differences do not really exist.

One purpose of the present research was to investigate as fully as possible the relations existing between the lines of a band and between the several bands, taking into account those groups above

[1] *Ann. de Chim. et Phys.*, **7**, 5–102, 1886.

[2] *Proc. Amer. Acad.*, **14**, 62, 1878. [3] *Madeira Spectroscopic*, 1881.

[4] *Proc. R. S.*, **54**, 200; also *Astronomy and Astro-Physics*, **12**, 547, 1893.

[5] *C. R.*, **100**, 854, 1885. [6] *Phil. Mag.*, **26**, 286, 1888. [7] *C. R.*, **100**, 854, 1885.

a which do not seem to have been considered before. In order to do this satisfactorily it was necessary to have the wave-lengths measured very accurately, and on examination it did not appear that there had been made by any one observer a complete and accurate measurement of the whole spectrum. Cornu,[1] in 1886, made extensive measurements on A, B, and *a*, but with his apparatus, although he had a good grating, it was difficult to get a precision greater than 0.02 where unity was 10^{-6} mm; whereas at present it is possible to measure many of the lines with an accuracy greater than 0.01 where unity is 10^{-7} mm, *i. e.*, with a precision twenty times as great. Cornu's error would be 0.2 of a unit on Rowland's scale, an amount, as will be seen later, almost equal to the total variation upon which Deslandres' first law is based. Consequently, Cornu's measurements were not available in the present work. The best determinations previously made are those of Rowland and Higgs, but neither gives all the lines even of the groups A, B, and *a*. Rowland's measurements are nearly complete for B, but he gives few for the other groups. Higgs gives A and B and up to the ninth pair of *a*, and although he and Rowland agree remarkably well in general upon B, judging from the few lines in A which both have measured, the agreement is not so good, there being much greater discrepancies than one would expect from the accuracy claimed for their measurements. Hence it seemed worth while to make new determinations of all, or nearly all, the lines previously measured; in addition many are given which have not been measured before, so far as the writer is aware. It is hoped, therefore, that the present determination of the wave-lengths, taking into account the best previous results, has both extended and unified the measurements on these bands and rendered them on the whole more accurate, thus doing for the absorption spectrum of oxygen what the similar work of M. Eisig[2] has done for the line spectrum.

Because of the precision which it is possible to obtain in the measurement of this spectrum, a careful study of the relations subsisting between the lines and bands furnishes an excellent test of the

[1] *Ann. de Chim. et Phys.*, **7**, 5–102, 1886.

[2] "Das Linienspektrum des Sauerstoffs," *Wied. Ann.*, **51**, 747, 1894.

exactness of the so-called laws of Deslandres[1] for band spectra. They may be stated briefly as follows:

I. In a given band the intervals from one line to the following in any series, calculated in vibration-numbers, are in arithmetical progression: *i. e.*, the lines are connected by a relation of the form

$$1/\lambda = N = a + bn^2,$$

where a and b are constants and n takes on all integral values from 0 to n.

II. When two or more series arise from the edge of a band, they are similar in all respects, and all bands belonging to the same substance have the same number of series.

III. In a series of bands the vibration-numbers of the edges form a series similar to that of the lines in a single band, or

$$N = A + Bm^2.$$

These laws are the most general in their application that have yet been announced. Deslandres tested them upon many spectra, including the subject of the present study, but he has not published details showing the exactness of the agreement which he obtained.[2] Kayser and Runge[3] have obtained a general confirmation upon bands of many substances, including those of N, C, CO, CN, and I, but the laws do not apply equally well to all cases, and occasionally appear to degenerate into mere interpolation formulæ.[4] The difficulties in the way of obtaining more exact expressions for the laws are in measuring the wave-lengths of bands accurately enough to warrant taking account of small variations in the reciprocals, and, in the case of the third law, in finding a long enough series of bands capable of precise measurement. The most exact measurements by which the third law has been tested are those of Kayser and Runge on the band spectrum of CN. But here the longest series has only four bands, which gives but two second differences. Longer series

[1] "Caractères principaux des spectres des lignes et de bandes. Considerations sur les origines de ces deux spectres," *C. R.*, **137**, 1013, 1903.

[2] KAYSER, *Handbuch der Spectroscopie*, **2**, 474.

[3] "Ueber die Spektren der Elemente," *Abhandl. der Berl. Akad.*, 1888–92.

[4] An example in point is cited by KAYSER (*Handbuch der Spect.*, **2**, 479), in the case of the cyanogen band λ 3883.55. This band, however, Deslandres claims to be exceptional.

are obtainable as in the second part of the *N* spectrum from 500 $\mu\mu$ to 2.80 $\mu\mu$ cited by Deslandres in support of his third law, but the measurements are not so exact and the confirmation is only general. Moreover, most of the bands hitherto measured are in the upper part of the spectrum, where a small error in λ causes a large error in the reciprocal.

MEASUREMENTS OF WAVE-LENGTH.

In the measurements of Rowland and Higgs, to which reference is made above, both used the same unit, viz., 10^{-7} mm, and their results appear to be equally accurate. For those lines in B and *a* which both have measured, they agree in general to within 0.01 or closer. Whenever such agreement occurs, the value for the line adopted in the present work is the mean of the two. In some instances the disagreement is greater than one would attribute to errors of observation, and in such cases the value adopted is the mean of my own final result, and the one which it confirms, provided such agreement is decidedly stronger with one than with the other. In some cases the mean of all three measurements was taken. For the large majority of the lines in A and *a* the values given are the results of my own and Higgs' measurements alone. Those for *a′* and *a″* have not been given before. Since 0.01 of a unit is about the limit of accuracy in general, it has been thought best to retain only two decimal places in the wave-length, except in the case of B where many of the lines are taken as Higgs and Rowland give them. It may be added further that the third decimal place of λ rarely affects even the seventh place in the reciprocal.

In respect to the group A, Rowland and Higgs rarely agree more closely than several hundredths. The variations in their measurements of the same lines are anywhere from about 0.01 of a unit to more than 0.1, and are not systematic, and consequently not due to constant errors. These differences are perhaps not so surprising when we consider the difficulties in obtaining good photographs of A and the variable appearance of the lines with the state of the atmosphere. However, with good photographs, such as may be obtained with the best apparatus on clear, calm days with a high Sun, it seems possible to obtain measurements by independent observers which seldom

differ for any line more than 0.02 or 0.03 of a unit from their mean. Assuming this mean to be correct, none of the values given for the lines of the A group should differ by more than this amount from their true value, and most of them should be closer.

My own measurements of the A band were made from photographs taken with a large concave Rowland grating having 20,000 lines per inch and 21.5 feet radius installed in the Sloane Physical Laboratory. A clear, sharp photograph of the second spectrum of the ultra-violet was obtained in a narrow strip through the middle of the same plate, thus affording an excellent method of comparison with Rowland's standard lines in this region (about 3850).

The measurement of B and *a* is much easier, because the lines are more definite and there are plenty of well-defined standard lines between which to make interpolations. The chief difficulty is with the very weak lines of the last pairs, for which almost any magnifying power of the micrometer is too great. Some such lines were measured by first putting a very fine mark on the back of the plate coincident with the line as seen from above. The measurements of the negatives were checked also by micrometer measurements on Rowland's maps, which gave very good results, care being taken to set on the center of density of the lines; and in the case of very faint lines the latter method is the better. Interpolating between standard lines obviates errors in the map-scales which are considerable. The precision of measurement for the final values of these two groups should be at least equal to 0.01.

The group *a′* was first noted by Jewell.[1] Many of its lines are too faint to be measured directly with a micrometer either on the negatives or on the maps. For these a faint mark was used as before. In the "head" or first band of the group many of the lines appear double, and some foreign lines of the same intensity seem to be present. In making out the series for the "head," the line 5789.40, which is the "chief line," corresponding to similar lines in A, B, and *a*, has been assumed double, as it is in all other cases. The only indication of duplicity actually shown is its greater intensity and a certain flatness in the intensity-curve characteristic of close doubles. Moreover, the regularity of the series calls for a double

[1] *Astronomy and Astro-Physics*, **12**, 815, 1893.

here. The accuracy of measurement for a majority of the lines is about 0.02 tenth-meter.

The positions of many of the lines in a'' were calculated approximately from relations established between the other bands. The observed values differ by less than 0.2 of a unit from those calculated. The lines are all extremely weak. Some, though not all of them, appear on negatives taken in zero weather, which indicates that they are not water-vapor lines. The first band of the group begins as usual with a double line, possesses a chief line, and a final pair in its proper position, as a glance at Figure 1 will show. Probably not all the lines present can be seen. Many are so faint as to be visible only on the charts, and then only when they are held in certain positions with respect to the light, or in such a position that the eye gets the effect of increased density by looking along the line. Some of the negatives were enlarged on ordinary sensitive plates, and two such enlargements superposed, but this did not bring out all the lines. Two or three lines are stronger on the corresponding chart of Rowland's first series, which is considerably more intense, though lacking in definition. No attempt has been made to measure most of the lines of this group nearer than to the nearest half-tenth, nor to arrange the lines in series.

Blunders and mistakes in calculation for all groups except A have been practically eliminated by the use of verniers made to fit Rowland's charts. The verniers were arranged to read directly to 0.04, and by estimation even closer; and, in spite of irregularities in the map-scales, any but very small mistakes could be detected at once. The wave-lengths for the several groups are given in the accompanying table.

The terms "head" and "tail" or "train" used to designate the two parts of the A, B, and a groups cannot be taken in this case in the usual sense of these terms as applied to band spectra, and are really misnomers. The spectrum is composed of two series of entirely separate bands instead of a single series, the so-called "heads" forming the first and the "tails" the second. The first series has the appearance of being nearly all "head" and the second all "tail," but the apparent crowding and confusion in the case of the former is due to the distance between the first few pairs being less than their

width. In all these bands the distance between pairs becomes greater with increasing wave-length, while the width of the pairs becomes less, and these two changes acting in the same direction

TABLE I.

NOTE.—Each band contains two series, which are arranged in pairs. Consequently, to obtain a single series alternate numbers must be taken.

A		B	
First Band	Second Band	First Band	Second Band
7594.00	7621.27	6867.458	6884.080
95.27	23.53	68.457	86.004
94.28	24.77	67.794	86.982
95.55	27.30	68.780	89.183
		(double ?)	
94.81	28.52	68.337	90.144
96.06	31.28	69.338	92.614
95.55[1]	32.49	69.144	93.559
96.79	35.47	70.130 [3]	96.282
96.51	36.65	70.220	97.197
97.74	39.86	71.180	6900.196
97.70	41.01	71.528	01.116
98.90	44.46	72.489	04.363
99.14	45.59	73.078	05.263
7600.30	49.27	74.039	08.785
00.80	50.40	74.888	09.677
01.95	54.33	75.830	13.449
02.65	55.45	76.953	14.331
03.80	59.62	77.878	18.365
04.73	60.73	79.275	19.245
05.87	65.14	80.173	23.542
07.05	66.25	....	24.416
....	70.89	....	28.986
....	71.97	....	....
08.20	76.86	....	29.839
09.57	77.92	....	34.669
10.72	83.06	....	35.518
12.33	84.11	....	40.584
13.45	89.47	....	41.430
15.32	90.50	....	46.770
16.41	96.11	....	47.580 ?
....	97.13	....	....
....	7703.02 [2]	....	....
....	04.02	....	....
....	10.16	....	....
....	11.16	....	....
....	17.60	....	....
....	18.55	....	....

[1] Higgs gives also 7695.42 and 7695.66. Probably outside edges of this line.

[2] These lines are taken from Higgs' measurements, but, judging from the uniformity of the preceding part of the series, they are a little large.

[3] The close double called the "chief line."

soon reduce the appearance of the first band to a regular arrangement of pairs like the second. That the "head" and "tail" are really separate bands is apparent from the following considerations.

Speaking generally, relations subsisting between the lines and bands of the first or "head" series, analogous to those between the lines and bands of the second or "tail" series, are always of a differ-

TABLE II.

a		*a′*	
First Band	Second Band	First Band	Second Band
6276.81	6287.94	5788.33	5796.30
77.66	89.60	(88.55)[4]	97.76
77.03	90.42	89.00	98.43
		(double ?)	(covered)
77.86	92.35	88.75	5800.18
77.52	93.15	89.40	00.83
		(chief line)	
78.29[1]	95.36	89.40	02.87
78.29	96.14	(89.71)	03.51
79.07	98.64	90.07	05.84
79.31	99.41	90.32	06.47
		(double ?)	
80.08	6302.18	90.97	09.10
80.61	02.95	91.49	09.72
81.37	06.00	(91.78)	12.64
82.16	06.75	92.15	13.25
82.93	10.06	92.96	16.46[5]
....	10.81	93.60	17.07
84.00	14.40	....	20.58 [6]
84.75	15.14	....	21.16
....	19.02	....	24.94
....	19.75[2]	....	25.52
....	23.92	....	....
....	24.64	....	....
....	29.10	....	....
....	29.82	....	....
....	34.55	....	....
....	35.26[3]	....	....
....	40.28	....	....
....	40.98	....	....
....	46.27	....	....
....	46.96	....	....

[1] "Chief line" and evidently a close double. Higgs gives also 6278.19 and 6278.38, apparently the outer edges.

[2] End of Higgs' measurements. [3] Hidden by adjacent heavy line.

[4] Lines in parentheses do not appear to fit into the series. Perhaps foreign.

[5] Hidden by heavy adjacent line. [6] Very dim and hard to measure.

TABLE III.

a''	
First Band	Second Band
5377.20	5384.27
77.32	85.45
78.00	86.05
78.38 (chief line)	87.50 (double)?
79.45 (double?)	88.10[1]
80.00	89.85, 90.45 [2]
80.20 (double?)	92.55
80.85 (double?)	93.10[1]
81.40	95.55 (covered)
81.97	96.10

ent order of magnitude and are frequently different in sense. Both "head" and "tail" begin with pairs of almost the same width, which decrease in width with increasing wave-length at slightly different rates. No series in a "head" or "tail" is a continuation of a series in the other, as it should be if they were parts of the same band. In fact, if the head series be continued into the tail, or the tail series extended upward into the head, the calculated pairs of either fall irregularly between the observed pairs of the other. Also the first and second differences between homologous lines in the heads and tails form entirely different series, as do the ratios of the same lines. Further, while there are no lines in the places calculated for the tail series extended upward, faint lines appear to be in their proper places for an extension of the head, just as if the first band, instead of fading out gradually as the second does, should drop very suddenly in intensity on approaching the region occupied by the other. That this is apparently what happens is indicated also by the fact that the last line of what is usually considered the last pair in the "head" of B is scarcely half the intensity of its mate,

[1] Stronger on old series of charts.

[2] Hidden by a group of five heavier lines, none of which actually cover the positions, but the shading renders them invisible.

and in a is less than half. It is true that, in general, the first series of the "head" is slightly the stronger, but the change in the last pair is abrupt. A similar difference is noticeable in the corresponding lines of A and a', but it is not so marked. Continuing the series in the first bands of B and a, we find the following agreement between observed and calculated lines, the former being dim and somewhat nebulous, but capable of measurement from the charts closely enough to indicate an apparent connection with the band.

TABLE IV.

Continuation of First B Band		Continuation of First a Band	
Observed	Calculated	Observed	Calculated
6879.28 } Last pair strong lines	.....	6284.00 } Last strong pair	.....
80.17	.....	84.75	.....
81.80	81.85	86.09	86.11
82.72 (hidden)	82.72	86.88	86.84
84.65	84.67	88.48	88.49
85.54	85.52	89.20	89.20
87.75	87.74	91.14 (covered)	91.14
88.60	88.57	....	91.83
91.05	91.06	....	.....
91.87	91.87	....	.....
94.67	94.63	....	.....
95.50	95.42	....	.....

The physical characteristics of these weak pairs follow the general rule of the first bands in that the lines of the first series are slightly stronger. There are other dim lines between the main pairs, but they do not seem to have any connection with the series.

Indications that the first A band is continued beyond the last strong pair are not lacking, although they are not so good as in B and a.

Some of the lines in what Higgs calls the "secondary train of A," a series of sharply defined, less intense pairs situated between the main pairs, are in positions suited to the continuation of the head series; but most of them do not fit, unless we suppose the first band not only decreases in intensity on reaching the region occupied by the second, but also at first increases the width of its pairs and again decreases them, in harmony with the series of the second band. But the lines of this group are apparently still more complicated.

TABLE V.

Continuation of First A Band	
Observed	Calculated
7615.32 } Last strong pair	....
16.41 }	....
18.57	7618.53
19.51	19.60
22.05	21.97
....	23.02
25.63	25.64
26.73	26.65
29.38	29.53
30.55	30.51

In addition to the first "secondary train" of Higgs there seems to be a second one, which first makes its appearance just on the more refrangible edges of the lines of the sixth pair, and is visible for a few pairs farther, each succeeding pair being farther removed on the more refrangible side from the corresponding pair of the main series. The series cannot be observed far enough to decide whether it follows laws similar to those governing the others. Traces of similar series appear also in B and to a less extent in a.

Since band spectra are due to vibrating molecules or aggregates, we might expect these secondary series not only to occur, but also to be stronger and more numerous in the bands of greater wave-length which are due to the vibrations of the more complex molecules. The vibrations producing the secondary series would be in the nature of harmonics, and the fact that they are always more refrangible seems to indicate this. No attempt has been made to complete any of the secondary series in the present work, as not enough lines have been clearly identified. These series correspond to the less intense, more refrangible series observed by Deslandres in the spectrum of water-vapor.

In his study of the single band by means of the parabola, Higgs shows a smooth curve connecting the lines of the "head" and "tail" series, as if they were parts of the same band; leaving a gap, however, where the two parts are separated. But if the curve is drawn according to his specifications, it shows a fault or offset at the gap, just as we should expect from the foregoing, indicating that the series belong

to similar, but not to the same bands. Further, the distances in wave-lengths between homologous lines for any two "heads" form a decreasing series, while for the "tails" the series are increasing. In fact, from every standpoint the two bands of a group seem to be independent and the spectrum to be composed of two band-series.

The geometrical relations of the various groups are clearly seen in Figs. 1 and 4, which have been drawn from measurements of the wave-lengths.

The first line-series in the first series of bands is slightly stronger than the second, but the reverse is true in the case of the second band-series. Also the width of corresponding pairs decreases regularly from A to a''.

In the first band of B there are ten strong pairs, and in the second thirteen easily measured, with possibly two others which are too faint to admit of measurements accurate enough to decide whether they belong to the series or not. In the second band the rate of decrease in intensity changes suddenly at the eleventh pair. There are ten strong pairs in the "head," and from analogy we should expect a sudden decrease with the eleventh, and this is in harmony with what has been said above with respect to the continuation of the series.

The first band of a has eight strong pairs, and the second fourteen pairs. The same abrupt change in the intensity of the second band is noticeable in the eighth pair, though it is not so strong as the corresponding change in B. Likewise the lines indicating a continuation of the first band of a are not so weak as in B, again preserving the analogy. In A there are fourteen strong pairs in the first band and eighteen pairs in the second. In a' there are six in the first and nine in the second.

In most spectra it is the vibration-numbers which are subject to regular laws rather than the wave-lengths, but in this case it makes little difference which is taken so far as Deslandres's first law is concerned. This is due to the fact that none of the bands is very extended. The first law is roughly approximate in all the bands, but it does not hold to the degree of precision with which the measurements are made. The agreement between the observed values and those calculated by Deslandres' formula,

$$N = a + bn^2 \ ,$$

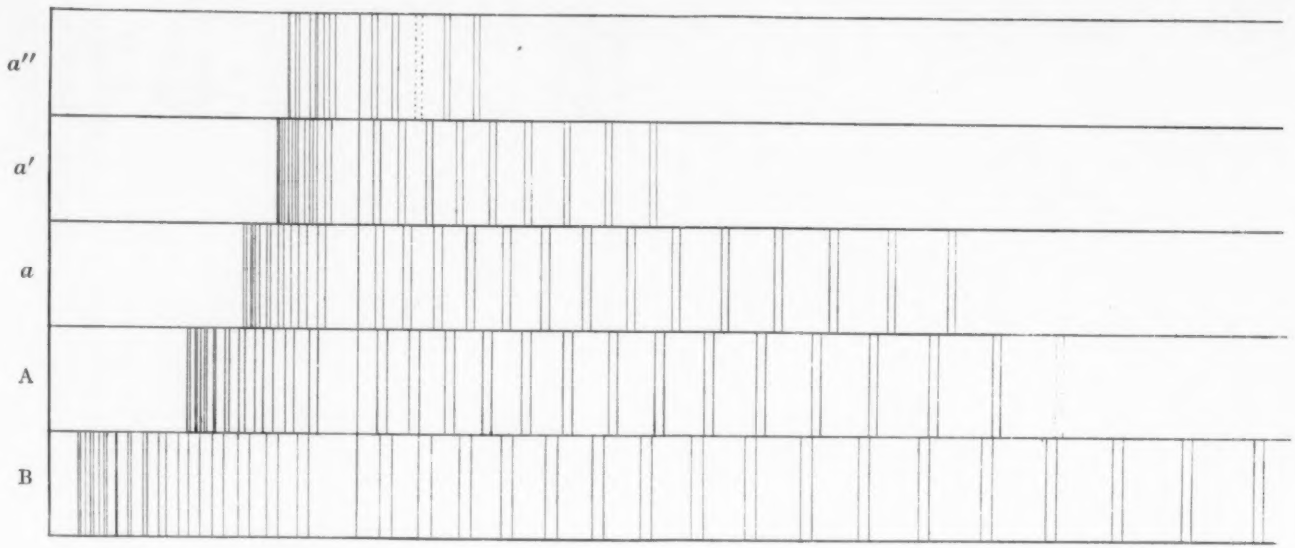


Fig. 1.—The groups as represented in Fig. 1 are drawn from the measurements of the wave-lengths. Beginning with A they are arranged in order, one above the order, with the first lines of the second bands forming a straight line. This pyramid-like grouping of the bands is especially effective in showing the symmetrical arrangement of the pairs from group to group.

is illustrated in the following applications to the first series of the first band of A and to the second series of the second band of B. The constant b is calculated from the sixth line in each case.

TABLE VI.

A, First Band, First Series			B, Second Band, Second Series		
N, Calculated	N, Observed	Diff.	N, Calculated	N, Observed	Diff.
13168.29	13168.29	0.00	14526.27	14526.27	0.00
168.03	167.81	+0.22	524.84	520.15	+4.69
167.26	166.89	+0.37	520.53	513.49	+7.04
165.98	165.61	+0.37	513.36	506.29	+7.07
164.19	163.94	+0.25	503.33	498.65	+4.68
161.88	161.88	+0.00	490.42	490.42	+0.00
159.06	159.39	−0.33	474.65	481.70	−7.05
155.73	156.51	−0.78	456.00	472.45	−16.45
....	....	......	....	....	......
....	....	...	....	....	......
142.65	145.70	−3.05	424.49	462.72	−38.23
....	....	......	....	....	......
131.37	136.58	−5.21	319.77	418.53	−98.76
124.96	131.43	−6.47	282.92	406.26	−123.34

If the law were accurate, the precision of the measurements would not allow a difference of more than a few hundredths in the last column, whereas they are generally several hundred times as great. The character of the variations plainly indicates that Deslandres' constant b is not really a constant, at least for this spectrum. A glance at the following values of b indicates this clearly. Two series from the B group are selected to illustrate the character and magnitude of the variations in the two band-series.

TABLE VII.

First series, first band, B		Second series, second band, B	
$b = -0.71$	when $n = 1$	$b = -6.120$	when $n = 1$
−0.465	2	−3.195	2
−0.397	3	−2.220	3
−0.366	4	−1.726	4
−0.345	5	−1.434	5
−0.330	6	−1.238	6
−0.321	7	−1.098	7
−0.314	8	−0.993	8
−0.308	9	−0.911	9
		−0.846	10
		−0.792	11
		−0.748	12
		−0.710	13

The variations are not only alike in general in the bands of the same band-series, but the values of b for homologous lines are approximately the same.

If the values of b are plotted as ordinates and the values of n as abscissæ, curves are obtained which at once suggest a much better law.

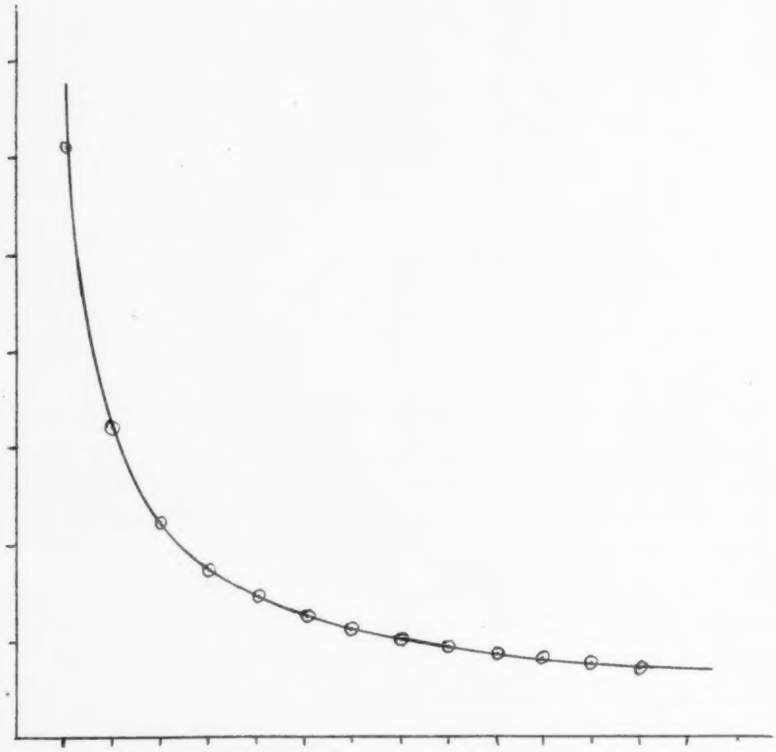


FIG. 2.—The bn-Curve for the Second Series, Second Band.

The two curves shown above are good types of all the bn-curves for the two band-series. Their form is very nearly an equilateral hyperbola, and assuming it to be such, we have

$$bn = k = const.$$

This, however, is not quite true, as k is still subject to a systematic variation which may be corrected as follows:

$$bn - \frac{n}{c} = k.$$

Here k is almost exactly constant, as may be seen by an application to the second series, second band, of B, and to the corresponding series of a.

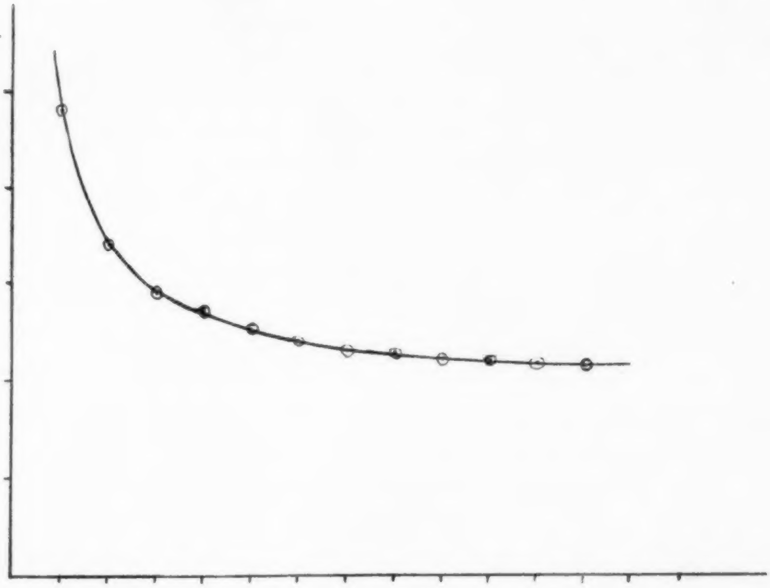


FIG. 3.—The *bn*-Curve for the Second Series, First Band A.

TABLE VIII.

$$bn-\frac{n}{c}=k$$

B	a
5.94	5.94
5.92	5.93
5.90	5.92
5.91	5.93
5.89	5.92
5.92	5.96
5.90	5.94
5.90	5.94
5.90	5.96
5.90	5.97
5.81	5.89

Hence we may write b as

$$b=\frac{k}{n}+c^{-1},$$

and substituting this value in Deslandres' formula, we have

$$N=a+kn+c^{-1}n^2.$$

That is, the correction for Deslandres' law is of the first order in n instead of being as usual of a higher order, the constant k being large in comparison with c. The constants c and k are different for the different series, but their variations are small.

The increased accuracy of the new formula is shown by its application to the same series of A and B calculated before (p. 95) by Deslandres' law. The differences only are given here corresponding to the third columns above. The formulæ used in calculation are as follows: for the A-series,

$$N=13168.29-0.305n-0.1953n^2;$$

and for the B-series,

$$N=14526.27-5.86n-0.2611n^2.$$

TABLE IX.

A, First Series, First Band			B, Second Series, Second Band		
Calculated	Observed	Diff.	Calculated	Observed	Diff.
		0.00			0.00
		−0.02			0.00
		+0.01			+0.02
		+0.01			+0.05
		0.00			0.00
		0.00			+0.02
		+0.04			+0.01
		+0.07			+0.01
		+0.04			−0.04
		+0.02			−0.07
		0.00			+0.01
		−0.04			−0.02
		−0.07			−0.05
		−0.11			−0.16

It is quite apparent that the proposed formula is much more accurate than the old one and gives results which agree with observed values to about the degree of precision of the measurements. The smoothness of the bn-curves and the smallness of the differences between the observed and the calculated values of N show that the

lines possess a very regular arrangement; and from the near approach to constancy observed in the second differences of both N and λ it is evident that some law based upon this fact is the true one. Deslandres' formula represents but one out of several possibilities, and, in the case of this spectrum at least, it is not the correct one. It is quite probable also that the proposed formula will be found to fit the line-series of other band spectra more closely than that of Deslandres, but this I have not as yet investigated. If the series were longer, it might be necessary to add another term, possibly one depending in some way on the wave-length. Assuming the law to be exact, we have, on the other hand, a good criterion of the accuracy of the wave-length determinations, and the above table of differences between the observed and calculated values would then show, in general, a greater degree of precision than has been claimed for them.

The new formula may perhaps possess some theoretical importance, inasmuch as in most discussions of a theoretical nature it is tacitly assumed that the first power of n does not occur, and systems are sought which can give rise to vibrations conformable to the formulæ of Balmer, Rydberg, and Kayser and Runge. In a recent contribution to the study of the structure of band spectra from a theoretical standpoint H. Nagaoka[1] discusses the oscillations of a system consisting of a ring of negatively charged electrons surrounding a larger positively electrified mass, and he shows that the vibrations of such a system can represent, qualitatively at least, the arrangement of lines actually observed in band spectra as well as explain other phenomena connected with them. But the formula at which he arrives is that of Kayser and Runge, which certainly would not fit this spectrum with nearly the accuracy of the one proposed. Hence it would seem either that the various band spectra are not capable of being represented accurately by a general formula, or that there is some fault either in the premises or in the logic of the theoretical discussions.

The series in the various groups are different, as they are also in the two bands of the same group. Even the two series of the

[1] "Kinetics of a System of Particles illustrating the Line and the Band Spectrum and the Phenomena of Radioactivity," *Phil. Mag.*, (6) **7**, 445, 1904.

same band are not alike, and consequently, while Deslandres' second law may be regarded as an approximation, it is not, strictly speaking, true. Considering the series in the different groups, we find that the actual distance between homologous lines of corresponding series increases with increasing wave-length, but that the rate of progression decreases; *e. g.*, the average rate of progression in A is about 0.22 of a unit, whereas in a' it is about 0.28; but the distance between any two lines of an A-series is much greater than that between the corresponding lines of a'.

That the two series in any band are not alike is evident from an inspection of the series formed by the widths of successive pairs. In both band-series the pairs are much wider at the beginning of a band, and consequently the first series must have an increasing rate of progression over the second. This difference in width is very evident to the eye when pairs far enough apart are considered. It is difficult to decide whether the variable rate affects one or both series, but the following table indicates that, in general, the first series has a faster rate of progression than the second:

TABLE X.

	A		B		a		a'		a''	
	1st B'nd	2d Band	1st B'nd	2d Band	1st B'nd	2d Band	1st B'nd	2d Band	1st B'nd	2d Band
$\overline{\delta\lambda}_1$	0.225	0.226	0.248	0.251	0.27	0.27	0.2625	0.2800	....	0.30 (about)
$\overline{\delta\lambda}_2$	0.2216	0.227	0.245	0.250	0.27	0.268	0.2625	0.2801	....	....

$\overline{\delta\lambda}_1$ denotes the average rate of progression of the first series.
$\overline{\delta\lambda}_2$ denotes the average rate of progression of the second series.

Also, of the two band-series the second has the greater rate of progression.

The third law holds more nearly for the second series of bands than for the first, but even in the case of the former there seems to be a steady increase in the second differences from A to a''. There are not enough bands in the series to warrant an attempt at a correction of the formula. The third law holds approximately also for any homologous lines, but the variations from band to band are irregular.

From Figure 1 we should expect that if the upper bands were magnified in some proportion to their wave-length, they could be superposed exactly upon the lower bands. This cannot be done, however, except very roughly. In the first place, the rate of progression of the upper bands, being larger, would be magnified also, and consequently only a limited portion of any two bands could coincide. If two bands have such a relation that one could be superposed upon the other, the ratios of their homologous lines should be constant; and they should be constant, or at least admit of very slight variation, even in the sixth decimal place, as that place affects hundredths in the wave-lengths. In reality, the ratios B/A and B/*a* for the homologous lines of any two series are not constant even to the fourth place. The ratios in the first band-series of A and B form a steadily increasing series, while similar ratios for the second band-series form a series which decreases regularly to a minimum at the fifth pair and then increases in the same way to a maximum at the end. The ratios between B and *a* give series which first increase and then decrease, the maximum occurring at the fifth pair, where the minimum occurs in the ratios of A and B. Similar ratios between B and the other bands show peculiarities which add to the complexity of the relations of these groups.

The ratios increase more or less regularly from band to band.

$$\frac{B}{A}=0.903+ \qquad \frac{B}{a'}=1.186+$$

$$\frac{B}{a}=1.094+ \qquad \frac{B}{a''}=1.277+.$$

The question of the extent of the bands of this spectrum and of the occurrence of this particular type of band in other spectra is an interesting one. Liveing and Dewar,[1] in their study of the oxygen absorption spectrum, with the gas confined in high-pressure steel tubes, observed bands corresponding to A, B, and *a*, and in addition a diffuse band with maximum intensity at about λ 5785, which agrees with *a′*, solar, at 5788; also a faint narrow band at about λ 5350, which is approximately the position of *a″* (λ 5377). They observed further, a strong band extending from about λ 4795 to λ 4750 and a very faint one at λ 4470. The last two bands could not be found in

[1] *Chemical News*, **58**, 163, 1888.

the solar spectrum because of the many fine lines in that region, although, according to Liveing and Dewar[1], Ångström is supposed to have seen in the solar spectrum at times of intense cold all the bands observed by them except the last. However, if they are present, they cannot belong to the same series as the lower bands.

No investigation yet made in the infra-red region is sufficiently accurate to determine whether other bands of the series exist below A or not. The next band, A′, would fall at λ 8510.5 and A″ at λ 9701.2. It is interesting to note, however, that in Langley's[2] bolometer chart there is a group corresponding to the position of A′ and very similar in general appearance to A, having heavier and wider spaced lines, as we should expect. Not enough detail is given to enable one to decide with certainty. From Figure 4 we may get an idea of the appearance of such a band, did it really exist, as well as of other bands toward the violet. The occurrence of similar bands in the ultra-violet spectrum of water-vapor has already been mentioned, and considering the cause of band spectra, it would seem that there is a tendency to produce this type of band also in those compounds in which oxygen forms so large a part of the molecule or aggregate as to impart its own characteristic vibration to the whole.

The points of chief importance in the foregoing discussion may be summarized as follows:

1. The general accuracy of the determination of the wave-lengths of the groups A, B, and *a* has been greatly increased and the series which compose these bands considerably extended.

2. The band *a′* has been measured and its relation to the other groups studied for the first time, and in addition a new group *a″* has been observed and studied at λ 5377.2.

3. The oxygen absorption spectrum has been shown to consist of two distinct series of bands instead of one, the series of bands occurring in pairs just as do the series of lines in a band.

4. Deslandres' first law is shown to be entirely inadequate to represent the line-series of the several bands, and a modification is proposed which gives results agreeing with the observed values to about the degree of precision of the measurements.

[1] *Loc. cit.* [2] *Ann. Astrophys. Obs. Smithsonian Inst.*, 1, 1900.

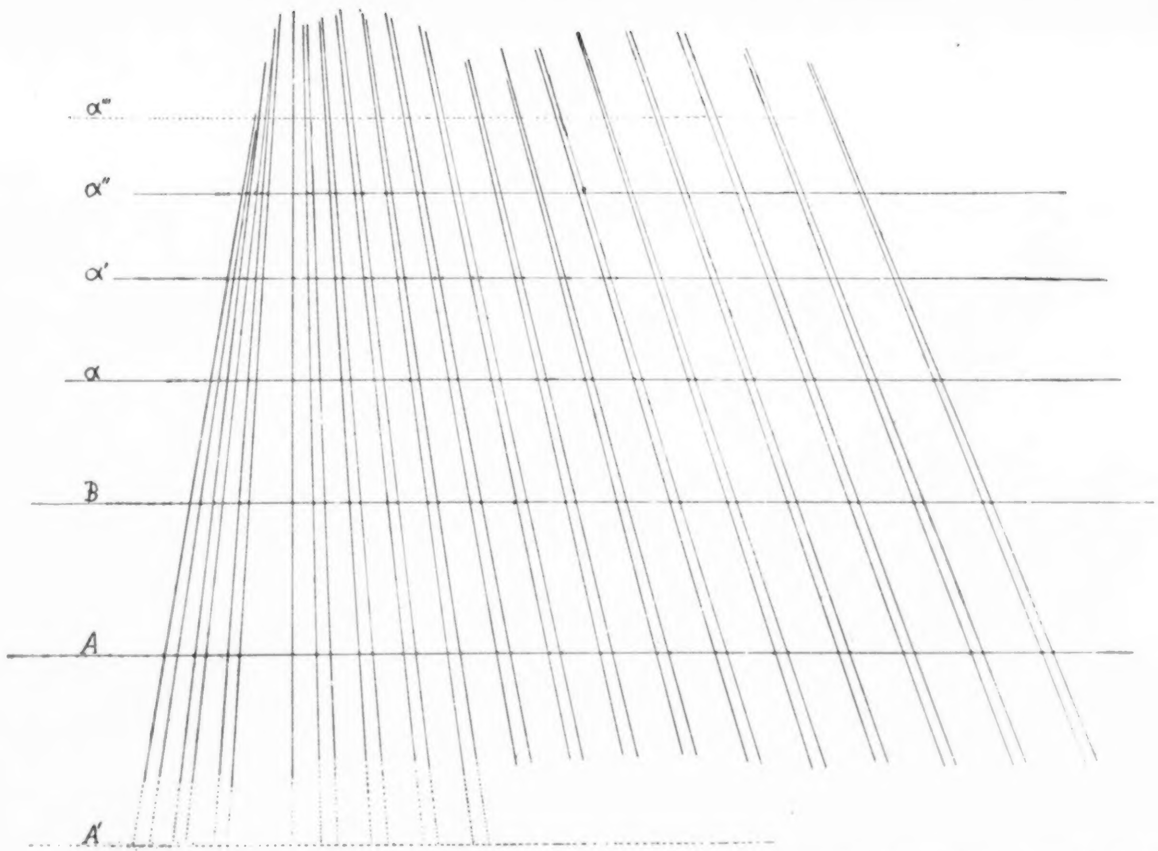


FIG. 4.—The distance between the lines marked A and B is 2 cm, representing the distance in wave-lengths between these two bands. The other horizontal lines are drawn at distances proportional to their distance in wave-lengths from A. Plotted in this way, the homologous lines of the two band-series fall almost exactly on straight lines, this being especially true for those lines whose wave-lengths are most accurately measured.

In conclusion, I wish to express my thanks to Professor A. W. Wright, whose kindly interest and criticism have been of great benefit throughout this investigation, and through whose aid the excellent photographs of the spectrum were obtained.

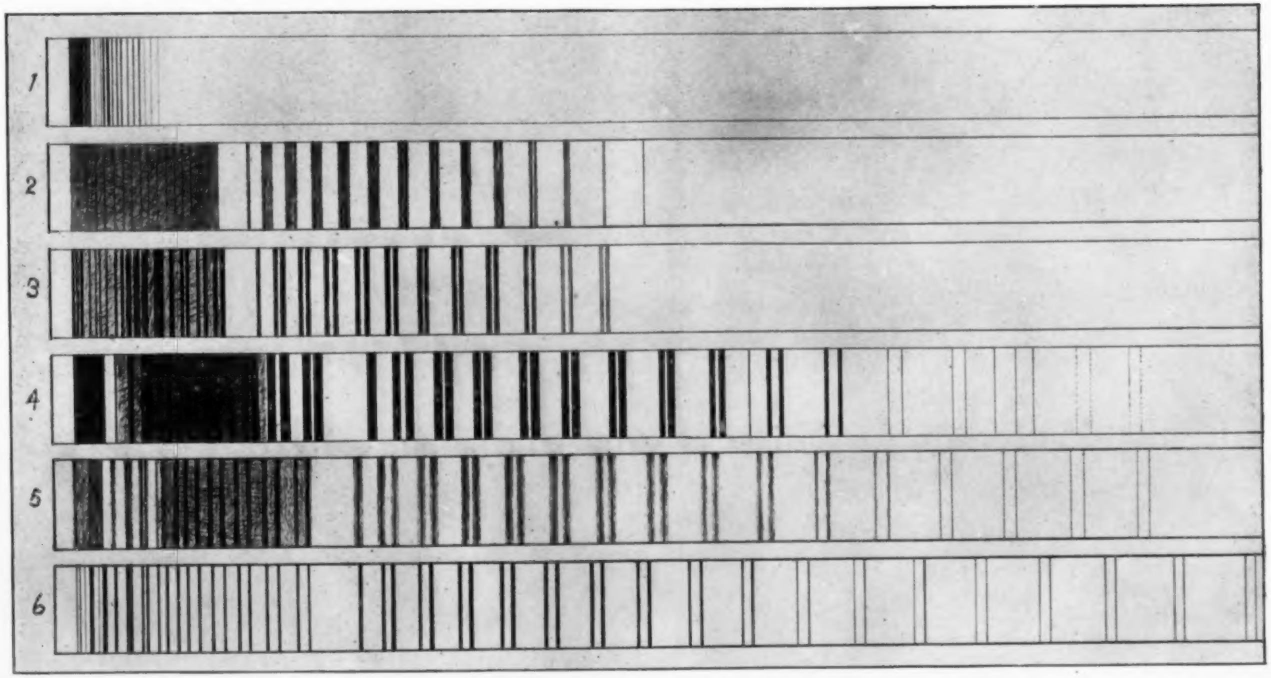
SLOANE PHYSICAL LABORATORY, YALE UNIVERSITY,
July 8, 1904.

EXPLANATION OF PLATE VI.

The drawings in Plate VI show the appearance of the A group as seen by various observers, and also illustrate the progress in the development of more powerful and perfect spectroscopic instruments. The bands are not all drawn to the same scale, which would have been very desirable. It was impossible to do this in some cases. The drawings have all been made for a high Sun, so that the lines appear as sharp as possible.

1. A group; Kirchhoff; prismatic spectrum.
2. A group; Thollon; prismatic spectrum.
3. A group; Langley; grating spectrum.
4. A group; Piazzi Smyth; grating spectrum.
5. A group; Cornu; grating spectrum.
6. A group; drawn from photographs with the Rowland grating in the Sloane Laboratory. The last three pairs are added from measurements of the wave-lengths. The eighteenth pair is cut off. The drawing was finally compared also with Higgs' excellent photographs of this group. All details are shown except the "secondary series."

PLATE VI.



DRAWINGS OF THE A GROUP.

THE NEBULÆ IN THE VICINITY OF *NOVA PERSEI*.

By H. Seeliger.

I HAVE shown in an earlier paper,[1] which I shall designate in what follows by (I), that the phenomena exhibited by the nebulæ in the neighborhood of *Nova Persei*, according to the observations hitherto published, may be completely explained if we assume that they had their origin in the illuminating action by the sudden and brilliant outburst of the *Nova* upon cosmical clouds—this expression being used in its most general sense. Several papers published by others since my former article appeared have led me to return to the subject, as it is my opinion that they are calculated to obscure the substance of the matter, which I believe that I had established.

I shall depart from the same point in this discussion as in (I), but shall merely add a slight generalization. Let the *Nova*, N, be the origin of a rectangular system of co-ordinates, with the X axis extending in the direction toward the Earth, which may be situated at a distance ρ_0, and with the Y and the Z axes in a plane passing through N perpendicular to X. Further, let $r=\sqrt{x^2+y^2+z^2}$, and ρ be the distance of the point (x, y, z), from the Earth. The outburst of the *Nova* will be observed at the Earth later by the time $\frac{\rho_0}{v}$ than it actually occurred, if we denote by v the velocity of light. The time t being reckoned from the instant when the outburst was noted, a radiation proceeding with the velocity v from N will appear to have attained a distance r at the time t, if

$$t+\frac{\rho_0}{v}=\frac{r}{v_1}+\frac{\rho}{v}\ . \tag{1}$$

Accordingly the phenomenon will take the form that would be given if all the points lying on the surface (1) were at the time t encountered by the radiation, be this of any kind whatever. If this radiation consisted of an emission of luminous particles, the particles emitted from N at the time $t=0$ would therefore lie on the surface (1) at the

[1] This JOURNAL, **16**, 187–197, 1902; I would remark that certain *typographical* errors in that article were corrected by the editors in Vol. **17**, p. 163, 1903.

time t. The eruption of N evidently lasted only for a very short time Δt, of a few days, in such powerful intensity as to have produced appreciable effects at not too small distances r. All the particles which possess or receive an appreciable effect of radiation will accordingly appear to lie between (1) and a surface, whose equation follows from (1), if we replace t by $t+\Delta t$. The brightness of the particles situated within this thin stratum will depend upon r, along each surface (1). If the eruption at N was a phenomenon which could be designated as a "light explosion," then the surface brightness is proportional to r^2; otherwise it either depends upon r in the same manner, or it diminishes still more rapidly with increasing r. But even if we proceed further outward from N in the same direction through the thin stratum, we shall encounter particles of different degrees of brightness. For the explosion doubtless lasted several days, while the effective emission of light from N was very considerable after a sudden increase, and decreased slowly at first and then more rapidly later. It would not be difficult to take this fact into account, but such a complication would nevertheless have no significance in a general consideration intended for the purposes of orientation, since Δt is certainly only a small quantity. Therefore let it be assumed that the particles within our very thin stratum are equally bright in the direction of the radius vector.

The surface (1) is produced by the rotation of a curve of the fourth degree, familiarly known as the Cartesian oval. It would be entirely useless in the case before us to base our considerations upon the rigorous equation (1); indeed, the distance ρ is so great relatively to the dimensions of the nebulæ that we may certainly place $\rho=\rho_0-x$. Thus we get

$$t=\frac{r}{v_1}-\frac{x}{v},$$

or, if we further place $x=-r\cos\phi$,

$$r=\frac{v_1 t}{1+\frac{v_1}{v}\cos\phi}, \qquad (2)$$

which is the equation of a conic section with its focus at N, the eccentricity of which is $e=\frac{v_1}{v}$, and its semi-parameter $p=v_1 t$.

For $v_1 > v$ and for a negative t, equation (2) represents the branch of the hyperbola convex toward N; N is therefore the focus situated outside of this branch. The larger $-t$ becomes, so much more will this branch of the hyperbola be displaced forward in the direction toward the Earth with increasing p. Nebulous structure must therefore have been visible in the neighborhood of the *Nova* before it flared up, in so far as the conditions were present in the direction toward the Earth for the visibility of the effects of radiation. In any case, we could be concerned only with short intervals, and we cannot invoke the testimony of observations, since the necessary photographs of the neighborhood of the *Nova* having sufficiently long exposure and sufficiently high sensitiveness, shortly *before* it flared up, are not available.

It therefore seems to me a wholly erroneous procedure to exclude the case of $v_1 > v$ as inconsistent with the observations, on the basis of these considerations. But, *per contra*, we shall be obliged to do this for the reason that there are not known in physics either radiations or translatory motions of matter having greater velocities than that of light. We must not in astronomy deal with analogies which, though perhaps thinkable, have no foundation in experience! Furthermore, the case $v_1 > v$ presents in a formal way hardly anything different from the case $v_1 = v$, which I treated solely in (I), and which I also still consider to be entirely sufficient.

The phenomenon is most simply interpreted as the reflection of the light proceeding from N by an aggregation of matter in the neighborhood of N in a state of the most extremely fine distribution. In truth, this interpretation has only to do with facts of the most trivial character, considered qualitatively, and none of the mysterious assumptions come into question which we are otherwise forced to entertain. I would therefore beg to point out again that I discussed the question of the illumination of cosmical clouds, masses of dust, and the like by neighboring fixed stars long before the appearance of *Nova-Persei*, and that I demonstrated the possibility of nebulæ in the neighborhood of fixed stars becoming visible solely by the reflection of the star light. This discussion was published at a time[1]

[1] "Ueber kosmische Staubmassen und das Zodiakallicht," vorgetragen in der Sitzung vom 6 Juli 1901 der K. bayer. Akademie der Wissenschaften.

when it was not possible for me to know in any way that nebulous structure was photographed in the vicinity of *Nova Persei.* This remark seems to me of importance, because it shows that the reflection theory was not made *ad hoc;* but it was, on the contrary, developed from considerations totally independent of the appearance of *Nova Persei.* In that paper I reached the following conclusion, among others, from the computations made (pp. 275, 276). If we assume a star with an apparent magnitude of 10.4 and a parallax of $0\overset{''}{.}01$, then a dust cloud which is of itself dark, and is only illuminated by the star, will exhibit at a distance of *several* seconds an apparent surface brightness $c \cdot 10^{-7}$ times the mean surface brightness of the disk of the full Moon, c being a number which does not differ materially from unity. I then proved further that such surface brightnesses ought to be within the range of visual observation. The objections to the explanation of the events transpiring in the vicinity of *Nova Persei* as a result of reflected light must be regarded as *a priori* not well taken, when we consider that *Nova Persei* was shortly after its appearance some ten thousand times as bright as a star of magnitude 10.4; and when we further consider that plates having long exposures with the exceedingly powerful optical instruments, such as are employed at the Yerkes and Lick Observatories, must permit the detection of far fainter surface brightnesses than that assigned above.

In applying this reasoning to the case before us, it will be a wholly unjustifiable as well as an inadmissible limitation to assume that the cosmical clouds in the neighborhood of N were of homogeneous structure throughout. It would be equally unpermissible, and unfounded either on observed facts or on theoretical considerations, to assume that the radiation of the *Nova* itself should have proceeded with equal intensity toward *all* sides, as I emphatically pointed out in (I), and there considered in detail. In fact, I have no doubt that all the considerations which predicate uniformity for the radiation toward all sides must be dismissed as valueless and inapplicable.

We may now first follow further the assumption that the cosmical clouds surrounding the *Nova* exhibit points of condensation which are arranged according to surfaces or curves. We should then see the cross-section of this structure with the paraboloid (2) pertaining to the appropriate time t. I showed in (I) that for this case: (1)

Every brighter curvilinear region in the visible nebulous structure, as well as its alteration with the time, can be represented within wide limits by the reflection theory; and similarly for the motion of a bright spot. (2) This representation is possible for every assumed value of the parallax, whence it is impossible to determine the value of the parallax by the measurement of the photographic plates without invoking the aid of more or less arbitrary hypotheses. And I also added in (I) (p. 192): "It is hardly necessary to remark that the moving bright spot will retain almost the same form if the mass is distributed homogeneously along the wisp; otherwise its form must change." It would seem to me that this remark had been overlooked or misunderstood by others. The state of things is this: If a powerful outbreak from N took place toward a definite direction, and if the matter was pretty uniformly distributed for long distances in this direction, and exhibits no considerable condensations, then the phenomenon will be the same as if a luminous frustum of a cone continued in motion in the original direction with the velocity of light, like a large projectile having a base of any given form. The cone-frustum will change its dimension only in the direction perpendicular to r, and, in fact, proportionally to r^2. For values of r that are not too small, hence at larger distances from N, this increase of surface will be slight, and the form of the luminous spot will retain all of its characteristic features. In this respect the state of things is just as if the cone-frustum consisted of projected matter, except for the fact that small irregularities in the density of the cosmical cloud which the cone-frustum encounters permits an explanation of the changes in form and in motion—which is not readily possible in the case of the assumption that we are actually dealing with projected matter.

It has recently been asserted that by adopting what is in a certain sense a photometric point of view we may nevertheless arrive at a determination of the parallax from the measurement of the photographic images of the nebulæ. In consequence of the expansion of the radiation proceeding from N, and the attendant weakening of its effect, the nebulous regions can be followed only to a certain *maximum* distance r from N; and it is taken to be possible to find at every time t the nebulous matter which is situated at this maximum distance r from N.

If we disregard the difficulty, not to say the practical impossibility of fixing the apparent distances $\eta = \sqrt{y^2+z^2}$ from N of such nebulous regions corresponding to a maximum value of r, because they must be just at the limit of perception, and if we examine the relationships which will arise, we get the following result:

$$\eta^2 = r^2 - x^2 = r^2 - \left(\frac{v}{v_1} r - vt\right)^2 .$$

If it is now assumed that r is the assumed constant maximum distance, then

$$\eta d\eta = \frac{v^2}{v_1} (r - v_1 t) .$$

If we follow such a region along, the velocity $\frac{d\eta}{dt}$ will become constantly smaller with increasing t and η, and we could therefore determine the time t at which $\frac{d\eta}{dt} = 0$. From $r = v_1 t$ we could then find $\eta = v_1 t$, and if we could assign something like plausible values for v_1, the parallax of the *Nova* could be thence determined, as η was measured in seconds of arc.

This train of reasoning, however, makes use of an hypothesis of the class which I have designated as *arbitrary* in (I). Since the radiation proceeding from N can under no circumstances be regarded as equally intense in all directions, as shown above, the maximum r can in no wise be regarded as independent of the time, as the observed η applies at different times to different values of the angle ϕ. The condition $\frac{d\eta}{dt} = 0$ gives rather

$$0 = \left[r\left(1 - \frac{v^2}{v_1^2}\right) + \frac{v^2}{v_1} t\right] \frac{dr}{dt} + (r - v_1 t) \frac{v^2}{v_1^2} .$$

Here $\frac{dr}{dt}$ is wholly unknown, and the attempt to determine the parallax, even with the assumption for v_1, is wholly *misdirected.*

In spite of the great simplicity of the reflection theory, many astronomers, for reasons not known to me, have felt themselves called upon to imagine in the flaring up of the nebulæ near *Nova Persei* effects of radiations of other sorts, such as kathode rays, the emission of ions, etc., for which v_1 would be assumed as materially smaller than v. In (I) I have expressed my view that

this procedure is inadmissible, at least as long as the simple reflection theory has not been completely shipwrecked; that is to say, has come into obvious contradiction with the results of observations, which has thus far by no means occurred.

Nevertheless I should like to touch on one question which concerns the case $v_1 < v$. Curve (2) is then an ellipse with a semi-parameter $p = v_1 t$ and the eccentricity $e = \frac{v_1}{v}$. The major axis of the ellipse lies in the direction toward the Earth and the most remote points from N lie at a distance r_1 from N in the direction toward the Earth, where

$$r_1 = \frac{v_1 t}{1 - \frac{v_1}{v}} \ .$$

Hence we see that the greatest extent of this ellipse is not very large in comparison with p, if v_1 is materially smaller than v. The radius vector to the extremity of the minor axis makes with the direction of the negative X axis the angle ϕ_1, which is determined by the expression

$$\tan \phi_1 = -\sqrt{\frac{v^2}{v_1^2} - 1} \ ; \quad \text{or } v_1 + v \cos \phi_1 = 0 \ ,$$

whence ϕ_1 is independent of the time t. The luminous particles will therefore appear to fill up the space between the ellipsoids which correspond to the two values of the semi-parameter,

$$p = v_1 t \quad \text{and} \quad p = v_1 (t + \Delta t) \ .$$

The assumption of an equal intensity of the radiation in all directions, and of no lack of homogeneity in the cloud structure, now has as its consequence that the luminous nebulæ must appear as an accurate *circular disk* with N in the center. The radius of this circle is the apparent magnitude of the semi-minor axis b. This circular disk will not appear equally bright throughout, but it will, in general, fall off at first as we pass from the center to the edge, since we must anyhow assume that the intensity of the radiation emanating from N will decrease with increasing r. If we make the further, and certainly very plausible, assumption that the luminous nebulosity is so sparsely distributed as never to be wholly opaque, then the apparent surface brightness h at some point at an actual

distance r from N will be proportional to the thickness Δx of the stratum at this point, measured in the direction parallel to the direction toward the Earth. For orienting ourselves, let us assume the stratum to be perfectly transparent, when

$$h = f(r)\Delta x ,$$

where $f(r)$ denotes the dependence of the intensity of the radiation upon r. We shall assume that the whole stratum between the two ellipsoids is effective. Then we get by differentiation

$$\Delta x = \frac{vv_1\Delta t}{v_1 + v\cos\phi} .$$

For the extreme edge of the circular disk, however, $v_1 + v\cos\phi = 0$, *i. e.*, $\Delta x = \infty$; whence the edge must appear especially bright. We can of course carry out the computation more accurately and avoid the infinity, as well as to take account of any absorption that may occur; but the result remains the same in sense.

In any case, for smaller values of t, the edge of the circular disk would appear relatively bright; and under some circumstances this brightness would appear so conspicuous that, in addition to the fainter diffuse illumination around the *Nova*, there should appear a brighter circular ring with its center at N which expands proportionally to the time. If the luminous particles do not extend to the extremity of b, the ring disappears, and there remains only the circular disk filled with diffused light. It could hardly be asserted that the nebulæ near the *Nova* exhibited phenomena like these just described. There can be no thought of a radiation from N uniform toward all directions, even if we should wholly disregard an irregularity of the naturally dark masses in the vicinity of N, or of N itself.

With this is excluded also the possibility of determining the parallax, say by measurement of the apparent magnitude of the semiminor axis b, since it cannot be determined whether an appreciable radiation occurred in the direction then coming in question, which is determined by the angle ϕ_1. It could not therefore be asserted that the nebulous regions apparently most remote from N belong to the above mentioned angle ϕ_1, and the determination of the correct value of the angle ϕ_1 cannot be carried out.

MUNICH, June 7, 1904.

THE SILVER "GRAIN" IN PHOTOGRAPHY.

By Robert James Wallace.

ON THE SILVER "GRAIN" IN A DEVELOPED PHOTOGRAPHIC PLATE, WITH A CONSIDERATION OF THE INFLUENCE OF THE DEVELOPING AGENT AS MODIFYING ITS SIZE OR CHARACTER.

It is a matter of importance to those making use of photography as a means of recording scientific data that a definite understanding be arrived at regarding the size of the silver particles which constitute the image. More particularly is this so in the case of astronomical photography, where the original negatives must necessarily undergo considerable enlargement, in order that the detailed structure of the object photographed may be rendered readily apparent.

In many instances this enlargement is carried to such an extreme that the individual particles of silver composing the negative image become so obtrusive that detail is entirely masked (for close observation) and can be discerned only when the enlargement is held off at some distance. In such a case there is absolutely no gain but rather the reverse, as it is much easier to study detailed structure of any kind when at the distance of normal vision.

It is with special reference to this usage that the present work was begun, since there are many plates available for the astronomer or physicist, each of which is supposed to combine in itself (according to the manufacturer) those qualities which make them valuable, viz., speed, fineness of grain, and general uniformity.

Speed must necessarily be the first consideration, and this narrows down the number, so that in the present work those selected for test were as follows: Seed 27, "Gilt Edge;" Cramer "Crown;" Cramer "Instantaneous Isochromatic;" and Hammer "Special Extra-fast." In each instance the plates were taken from different emulsions, triplicate exposures being made for all results aimed for.

These makes were selected as being the fastest generally available, and tests for relative speed showed that the point of highest efficiency was about equally shared by the Seed "27" and the Cramer "Crown." The results of nine separate tests (from different emulsions of the

same trade brand) showed that, although the latter plate was *occasionally* a trifle faster, yet the Seed "27" was always of the same uniform speed, and gave the least amount of "chemical fog." The remaining two plates were somewhat lower in general sensitiveness, which may be proportionately represented as 9:11 between the "isochromatic" and "27", and 8:9 between the Hammer and "27," in favor of the latter.

It is very generally understood that silver bromide ($2AgBr$) is the chief substance employed in the making of gelatine dry plates, but that silver bromide exists in several different allotropic forms has long been known—the first, formed by the admixture of the gelatine and bromide and silver salts, is of low sensitiveness, but in the process of ripening passes gradually through several modifications, finally ending in a state which is capable of reduction by a developer without the previous action of light, viz., the blue allotrope of silver bromide.[1] If the ripening of the emulsion be stopped prior to the formation of this last form, the result is still another allotrope, which is green by transmitted light, and of high sensitiveness.

In the process of "ripening," which is brought about principally by an increase in the temperature of the emulsion, the introduction of ammonia, etc., it undergoes still another and purely physical change, viz., the particles of $2AgBr$ increase in size, in all probability due to accretion. The measurements of Eder give the particles in the finer ***unripened*** emulsion as from 0.0008 to 0.0015 mm, while in the most sensitive form he gives a size of from 0.003 to 0.004 mm.[2] As stated by Perrine,[3] the size in this latter state is approximately 0.0025 mm, while Kayser[4] places it at 0.001 mm.

Although a more sensitive emulsion certainly means a coarser grain, yet that coarseness of grain is not synonomous with speed is shown by the researches of Luppo Cramer, who instances the use of too strong a solution of nitric acid in the manufacture of the emulsion, or too strong ammonia (or too great a quantity), also an insufficient amount of gelatine; even the shaking up of the sediment during the process of "cooking" is likewise certain to result in coarseness.

[1]Blue by transmitted light.

[2]MELDOLA, *Chemistry of Photography*.

[3]*Amer. Annual of Photography*, 1904, p. 203. [4]*Handbuch der Spectroscopie*, I, 638.

Much has at various times been written regarding the value of certain developing agents as modifying the resultant size of these silver particles, and many claims have been advanced by the advocates of slow or dilute development as giving a negative with a very fine grain, and some diversity of opinion exists regarding the plate best suited for the work.

It goes without question that the plate best suited to the needs of the astronomer or physicist is that one which combines the highest speed with the requisite fineness of grain; for in the first case the higher the speed, the greater the efficiency of the telescope or instrument used; and in the second case the finer the grain of the original negative, the greater the available enlargement.

After consideration, the following method was adopted in making the tests as offering the least chance for error (as already mentioned), triplicate plates being made in every case, and at every point throughout the series.

An instrument was constructed upon the same lines as the sensitometer of Scheiner, consisting of a rectangular box open at either end and fitted with forty-two small rectangular cells. One end is closed by a sheet of thin metal in which is made a number of minute holes corresponding to the ends of the small rectangular cells, beginning with one and increasing in arithmetical progression. The plate under test was placed at the other end of the sensitometer and closed in light-tight. Exactly similar exposures were then made to ground glass illuminated by sky light.

COMPARISON OF SIZE OF "GRAIN" IN THE PLATES CONSIDERED.

Similar sensitometer exposures were made on each of the four makes of plates, as previously mentioned, the precaution being taken in the selection of the three plates of each that they should represent entirely different dates of emulsion. These exposures were then developed four at a time, the developing tray containing one plate of each "brand."

A hydrochinone + metol + adurol combination was selected as the developing agent, for reasons which will be obvious as this article progresses. Microscopic and photometric examination of the negatives showed results identical on each plate of the same make.

Photomicrographs were then made from equal opacity squares of one plate of each make, of sufficient magnification to show clearly the individual particles of reduced silver.

Considerable difficulty was experienced in this portion of the work owing to the definite thickness of the gelatine film on the plate and the consequent number of planes, which rendered focusing a matter of extreme care. The use of a lower-power objective could not be taken into consideration because the "grain" was not sufficiently resolved, an entirely false effect being thereby produced.

A magnification of + 430 (Fig. 1) being decided upon, the apparatus was firmly clamped and negatives made of the squares selected.

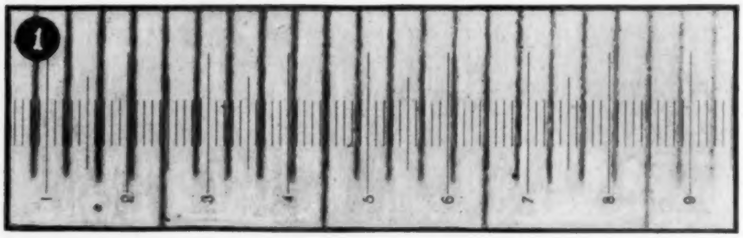


FIG. 1.—Scale of Enlargement for Photomicrographs.
4 divs. of ocular micrometer = 1 div. of stage micrometer;
1 div. of stage micrometer (ruling = 0.001 mm) = 4.3 mm; ∴ 1 div. ocular = 2.5μ.

At this point the triplicate plate exposure was of much assistance, for, as the field was entirely re-focused between each, the plates served as a check upon one another inasmuch as imperfect focusing brought into the field a new "set" of grains whose position would not coincide relatively with the accompanying two plates; to facilitate comparison and eliminate errors of personal measurement, an ocular micrometer was introduced, and brought to a focus at the plane of the sensitive plate.

It will be seen (Figs. 2–5, Plate VII) that the grain particles of the Seed "27" plate are decidedly the most regular of the series, while those of the Cramer "Crown" show the largest and most "ragged." It will also be noted that in the case of the isochromatic plate the appearance of the grain particles is altogether different from that shown by the others; for, while the general shape is more or less regularly round or spherical, the isochromatic grain is decidedly spicular.

PLATE VII.

Seed 27.
"Gilt Edge."
(1.4 to 1.8μ)

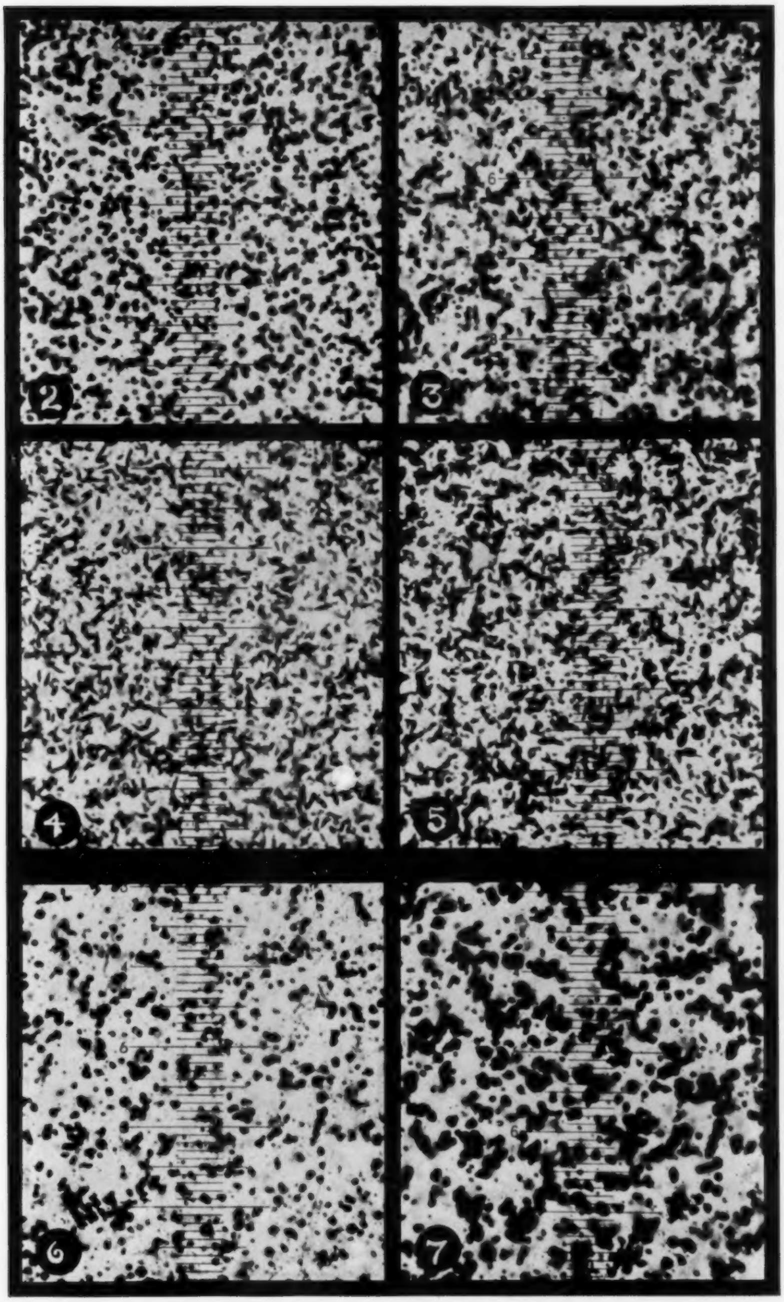
Hammer
"Special Extra Fast."
(3.5μ)

Cramer
"Inst. Iso."
(0.6 × 1.5μ to 1.1 × 3.7μ)

Cramer
"Crown."
(5.3μ)

Seed 27.
Before Intensification.
(1.3 to 4.0μ)

Seed 27.
After Intensification.
(2.8 to 10.4μ)



PHOTOMICROGRAPHS OF SILVER "GRAIN."

Examination shows that this spicular grain is not distributed equally throughout the depth of the film, but is almost entirely confined to the surface; for, whereas the film at this point is composed of grains of this character, they become fewer as the lower planes are successively observed, until at the lowest depth (in contact with the glass) they are entirely absent.

It must not be understood that these spicular "grains" gradually change shape and become irregularly round; they are a distinct set of particles by themselves, the gradual introduction of the others being readily seen with a sufficiently high power.

That this form of particle would seem to occur rather generally in isochromatic plates is shown by the fact that from a number of other "iso" plates, exposed and developed in the same way, and including Cramer "iso" plates of emulsions covering a period of eighteen months, Seed orthochromatic, Cramer trichromatic, and Lumière orthochromatic, the same spicular grain was noticeable in each, but varying in amount. The idea that this form of "grain" is inherent in and essential to an orthochromatic film the writer does not advance, there being several probable reasons for the formation, which it is purposed to treat to a further investigation.

Careful visual microscopic examination of these negatives also agrees well with the findings of Luppo Cramer, who states that each particle of Ag in the negative corresponds to a $2AgBr$ grain in the undeveloped layer, and that an increase in the exposure and development of the plate shows, first, "that the number of Ag grains in the upper layer is constant" (or nearly so); "second, the number of grains in a unit of volume increases; and, third, that the size of the individual grains increases;" and to this may be added a fourth point—that the grain particles become more complex by reason of their running together and forming what we may term as a *group-particle.*

ENLARGEMENT OF THE "GRAIN" BY INTENSIFICATION.

A sensitometer negative was firmly fastened upon the microscope stage and photomicrographs were made. The objective was then swung aside, and the square under magnification was intensified by mercuric chloride, followed by blackening with ammonia, the entire operation being performed by means of a small camel-hair brush.

When dry, the objective was replaced in position, and exposures were again made, which are reproduced in Figs. 6 and 7. The enlargement in size of each of the original particles is very marked, measurement of the "before and after" effect showing an increase from about 2.5μ to 5.0μ in the single grains, and from 4.0μ to 10.5μ in the group-particles. Observation will readily identify the individual grains in these two plates.

INFLUENCE OF DILUTION OR CONCENTRATION OF DEVELOPER, AND TIME OF DEVELOPMENT AS AFFECTING SIZE AND CHARACTER OF "GRAIN."

For this purpose a number of exactly similar exposures were made upon Seed "27" plates in the sensitometer and developed separately at the same temperature by different developing agents as follows:

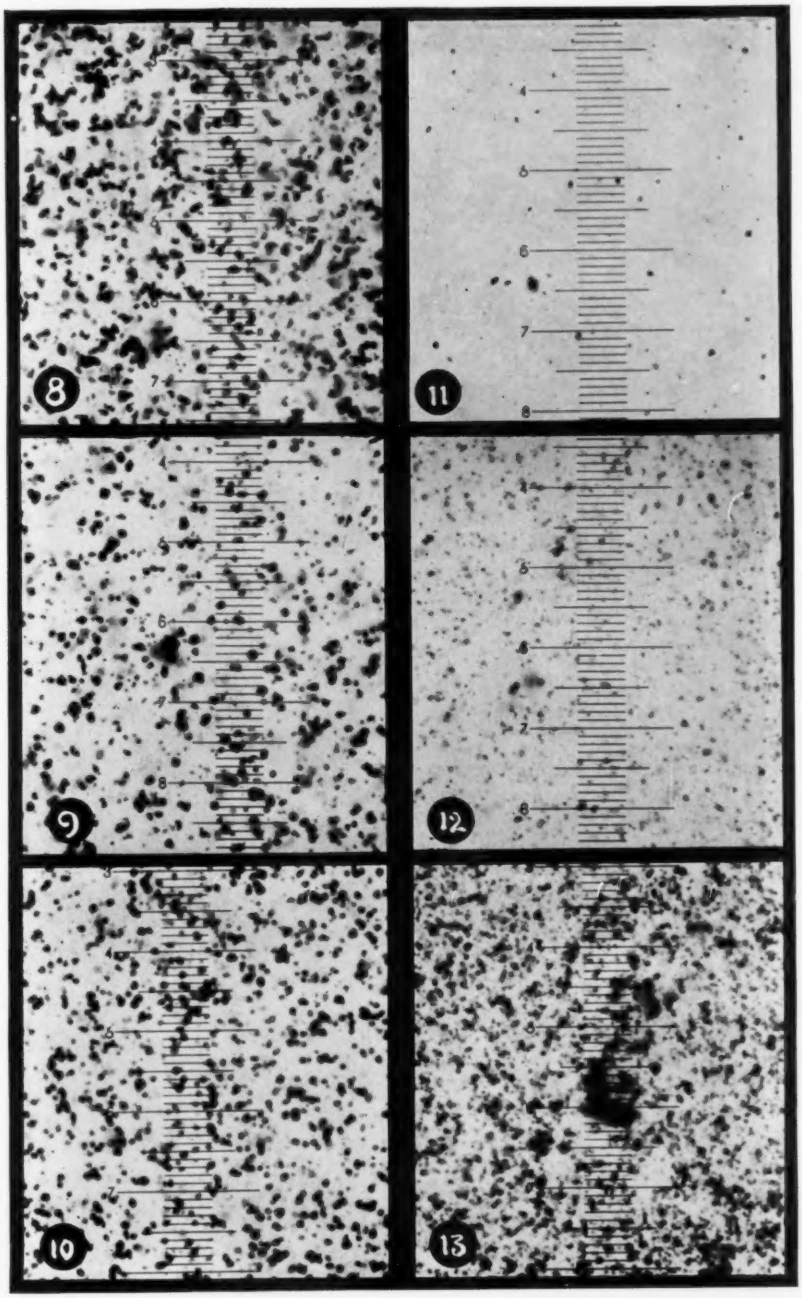
1. Rodinal. Development begun at 1:120 and continued for 15^m; successive additions of rodinal in single minims were made until the developing solution represented a strength of 1:40, taking 35 minutes more time. The 1:40 solution was then allowed to act for 10^m. Total time of development, 1^h.
2. Hydrochinone and caustic potash. Total time of development, 6^m.
3. Hydrochinone + metol + adurol + caustic soda. Total time of development, $1^m\ 20^s$.

These negatives were then dried in a current of air, and an equal opacity square selected from which photomicrographs were made.

A comparison of the results (Figs. 8–10) shows that in the case of the slow development by rodinal the character of the grain is vastly different from that of the remaining two plates, being decidedly more "ragged" in appearance, and showing an actual and definite increase in size, principally by reason of the running together of the several particles to form a new *group-particle.* In the case of the plate developed with hydrochinone, the "grain" is better, with less running together, while in that developed rapidly in the hydro-meto-adurol mixture the grains of silver are seen to be deposited in a much more definite and regular form than in either of the two preceding.

These results in the opinion of the writer, accord well with the theory of increase by accretion, in the length of development; for, according to Abney, when the silver bromide ($2AgBr$) is acted upon by light, there is first a chemical change to Ag_2Br (sub-bromide), followed by a physical change in the $2AgBr$ molecules, and the black-

PLATE VIII.



Seed 27.
Rodinal, Slow
Dev.
(3.0 to 8.7μ)

Seed 23.
Reduction
without exposure.
Dev. for 45
secs.

Seed 27.
Hydrochinon,
Medium Dev.
(1.3 to 2.4μ)

Seed 23.
Reduction
without exposure.
Dev. for 10 min.

Seed 27.
Hydro-Metol-Adurol, Rapid
Dev.
(1.4 to 1.8μ)

Seed 23.
Reduction
without exposure.
Dev. for 20 min.

PHOTOMICROGRAPHS OF SILVER "GRAIN."

ening consists, first, in a slight reduction of Ag_2 and continues by an interaction of the molecules in which the Ag_2 seizes upon the bromine in the adjoining molecule of $2AgBr$, and reduces that to a state permitting of reduction by the developer, while it in turn attracts the bromine of its neighboring molecule, and so on. Second, according to Luppo Cramer, the Ag ions give up their positive charge (caused by the impact of light) to the negative ions in the developer (which arise from the soluble salts in the developer), and the resulting supersaturated solution of metallic silver is deposited upon the nuclei of the Ag in the film; therefore, whatever theory be preferred, it naturally follows that the longer the action of development is continued, the greater will be the size of the particle, either by interaction or deposition.

The increase in the size of the individual grains was also very readily seen by developing a stained plate under the microscope. A drop of developer was applied while the $2AgBr$ grains were under observation, and an almost immediate reduction was observed. By repeating the experiment a number of times it was seen that the grains of silver bromide, when first acted upon by the developer, were reduced as individual grains of very small dimensions[1], only a portion of the $2AgBr$ particle appearing to be acted upon at first, but increasing in size individually and then coalescing into group-particles, which become larger as development is continued.

REDUCTION OF THE SILVER BROMIDE WITHOUT PREVIOUS EXPOSURE.

That there is a definite reduction of the $2AgBr$ particles in the film even when there has been no light impact, is well known, although not generally understood. The appearance of "chemical fog" upon the film is used by many workers as an indication of the point of maximum development.

That this "chemical fog" begins to be deposited almost from the instant of contact with the developing agent, was shown by a series of experiments as follows: Strips were cut (in darkness) from

[1]This is in agreement with LIESEGANG (*Archiv. für Wissensch. Photo.*, 1, 229), who says: "Since the advance of the reduction on to the unaltered silver bromide proceeds comparatively slowly, we must conclude that it is not always necessary for the groups of particles of silver bromide to be completely reduced. In this way a diminution in the size of the grain may therefore be possible."

a number of plates, including Cramer "Crown," Seed "27," Seed "23," and Carbutt "Lantern slide," and were then partially immersed in fresh hydrochinone developer for different lengths of time, from 5 seconds to 20 minutes, and passed immediately into the fixing bath. When dry, examination showed that reduction had taken place on all plates in amount varying according to the time of development. This reduction was relatively stronger on the "Crown" plate and weakest on the "lantern slide," the Seed "27" and "23" occupying places intermediate. The gain (by accretion) in the size of the grain particles is also very well shown in this series of plates. Figs. 11–13 (Plate VIII) give the record of the Seed "23" plate at 45 seconds, 10 minutes, and 20 minutes development, respectively. The exceeding fineness of the grain-particles in these plates will be noted, the size varying from 0.0017 to 0.002 mm mean diameter.

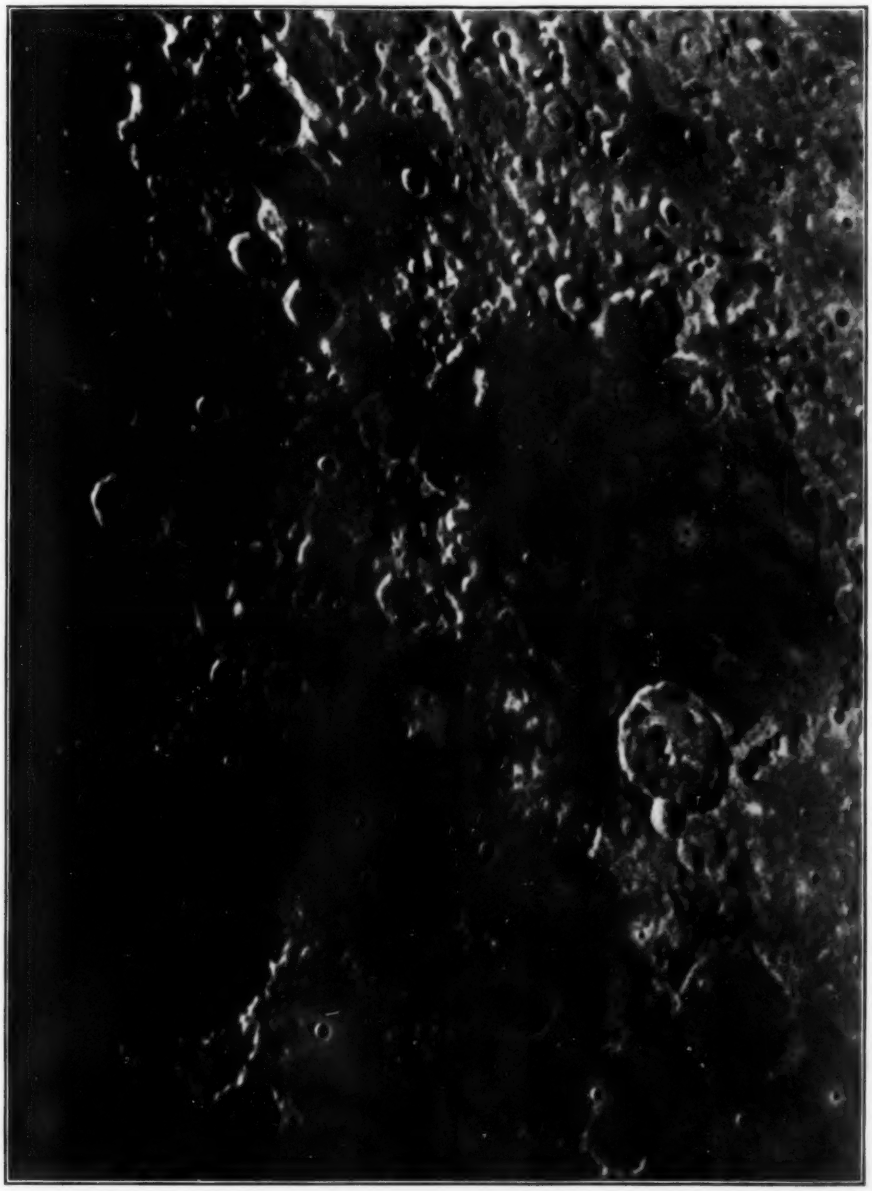
This reduction of the unexposed silver bromide was found to take place with all the developing agents tested, which include hydrochinone, hydro-eikonogen, metol, rodinal, adurol, and edinol. It would therefore appear that there is present in the film a certain amount of that allotropic form of $2AgBr$ which is capable of reduction without previous light-action, and which amount increases with the sensitiveness of the plate.

The idea generally accepted by a great number of workers that the size of the silver "grain" in a negative is dependent upon the size of the grains of silver bromide originally in the film; or, by still another class, that the slow or dilute development of the plate would give a negative whose "grain" would be distinguished by a particular degree of fineness, is not borne out by the results obtained and herein specified. On the contrary, we are led to deduce from these experiments:

1. That the original grain-particles of the silver bromide are by prolonged development considerably enlarged, by reason of the formation of *group-particles*, which are relatively enormously increased in size, so that a method of *rapid* development (provided the developer is compounded to give not too great a contrast) is the means of obtaining a more definitely uniform deposit of particles, which most nearly approach the size of those in the original $2AgBr$.

2. That of high speed American plates the Seed "27 Gilt edge"

PLATE IX.



LUNAR REGION NEAR GASSENDI.

Negative made with 12-inch visual telescope and color-screen. Diameter of original image, 2 inches; enlarged 11.3 times.

is, of the four makes tested, that having the finest grain-particles, of most definite uniformity; of equal speed with the "Crown," but with less tendency to "chemical fog."

3. That the intensification of the original negative should not be attempted where enlargement is to follow.

As an example of the efficiency of rapid development, attention is directed to Plate IX, which is reproduced from an enlargement of 11.3 diameters. The original negative was taken through a color-screen on a Cramer isochromatic plate; time of development, 70 seconds.

In conclusion the writer takes pleasure in acknowledging the kind assistance of Messrs. E. B. Frost and J. A. Parkhurst in much of the foregoing work, and also takes this opportunity to sincerely thank those who so courteously replied to his communication of the spring of 1903.

YERKES OBSERVATORY,
July 4, 1904.

NOTE.

The foregoing work was begun about the beginning of 1903, at which time the author sent out a circular letter to the departments of physics and of astronomy at a number of the leading universities and observatories throughout the United States, requesting data relative to the plates and developers in general use by them. The pressure of other duties prevented the completion of the work until July of the present year.

Upon completion of the paper, and after arrangements for publication, there came to the notice of the writer an article published by Messrs. A. and L. Lumière and A. Seyewetz entitled "The Influence of the Character of Developers on the Size of Grain of Reduced Silver" The method pursued and therein outlined by these eminent investigators, was as follows: Exactly similar exposures were made upon Lumière "blue label" plates of same emulsion, which were then developed by all the principal known developers (prepared normally and also with modifications) "until the images had reached a comparable density." The portion showing the greatest opacity was then selected from each negative and by the aid of hot water the gelatine was dissolved off. "This gelatine solution, well shaken, and

containing the reduced silver, was used for the preparation of material for microscopic examination." Photomicrographs were then made of the same magnification, and prints therefrom compared.

Among other conclusions thus derived, the investigators state:

"2. No apparent influence is shown in the size of the grain of reduced silver by temperature, concentration, or duration of development."

An evident discordance between this conclusion and that of the present writer, calls for a word of explanation. This apparent discordance would perhaps be easiest resolved if it were decided just what is to be regarded as the "grain" of the negative. In the opinion of the writer the general description "grain" is taken to mean the particles of silver reduced in the negative and *in situ*. It has been shown that the grain-particles of the $2AgBr$ are more or less modified in character, both by the method and duration of development, and the tendency of the grains to coalesce and form *group-particles*. A *group-particle* is undoubtedly formed of individual grains, but inasmuch as their units are now to be taken collectively as a new whole, they must be so considered. Granted even that the size of the individual grains remains unaltered by variation in development; yet, if a number of these particles be so welded together (as it were) by chemical or electrolytic action their *character* would be altered and hence necessitate a new consideration of their gross size.

R. J. W.

PLATE X.

S

STAR-CLUSTER *N. G. C.* 663.

Photographed with 40-inch telescope without color-screen. Enlarged 1.7 times from original negative.
Scale: 1 mm = 6.′1.

ON THE STELLAR PARALLAX PLATES TAKEN WITH THE YERKES TELESCOPE.

By Frank Schlesinger.

The success which attended Professor Ritchey's experiments in photography with the forty-inch Yerkes telescope, and the fine results obtained by him, have led to an attempt by the writer to use the telescope for the photographic determination of star positions, and especially for the measurement of stellar parallaxes. For such purposes the great focal length of the telescope, and the correspondingly large scale of the photographs, would seem to offer an unusual opportunity for a high degree of precision. Professor Ritchey's method was, in brief, to place a yellow color-screen immediately in front of a Cramer isochromatic plate, and to keep the latter stationary (as referred to the image made by the objective) by means of the late Dr. Common's double-slide plate-holder. This simple device of Common's not only makes an auxiliary telescope unnecessary, but insures guiding which is far superior to anything that could be done, in this case at least, by moving the entire telescope.

The screens employed by Professor Ritchey consisted of two approximately flat plates of clear glass inclosing a film of collodion and another of Canada balsam. Such a screen must introduce distortions other than those which are strictly geometrical. Whether these are great enough to interfere seriously with the delicate requirements of stellar parallax work is an open question. Although this disadvantage of the screen is probably not an insuperable one, it was thought better to avoid the difficulty rather than to devise methods for overcoming it. A little experimenting showed that for present purposes the screen can be dispensed with: that is, Cramer isochromatic plates at the visual focus give very good star images without anything between the objective and the plate. The accompanying illustration, Plate X, is from a negative of a loose cluster, *N. G. C.* 663, taken without a screen on October 11, 1903. The length of exposure was one hour, and the atmospheric conditions were good throughout. This negative was taken to show the fainter stars, and

consequently the half dozen bright stars are overexposed. A careful comparison of stellar plates taken with the screen and without shows that there is little to choose between them either as regards the minuteness of the images or their sharpness. This statement, which may seem surprising at first sight, is explained by reference to the accompanying diagram, showing the color-curve of the forty-inch objective and the curve of sensitiveness of the plate for different

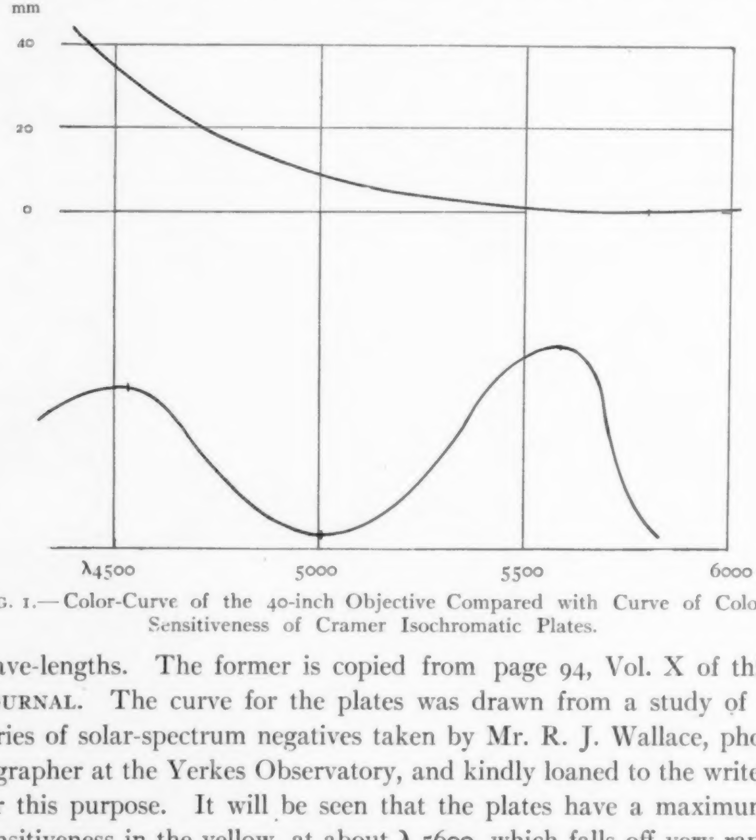


FIG. 1.—Color-Curve of the 40-inch Objective Compared with Curve of Color-Sensitiveness of Cramer Isochromatic Plates.

wave-lengths. The former is copied from page 94, Vol. X of this JOURNAL. The curve for the plates was drawn from a study of a series of solar-spectrum negatives taken by Mr. R. J. Wallace, photographer at the Yerkes Observatory, and kindly loaned to the writer for this purpose. It will be seen that the plates have a maximum sensitiveness in the yellow, at about λ 5600, which falls off very rapidly toward the red end, so that a prolonged exposure is necessary to show the D lines at λ 5890. There is a secondary maximum in the blue at about λ 4550, where the plate is nearly as sensitive as in the yellow. Between these two maxima, that is, in the green, the plates are only slightly sensitive, reaching a minimum at about λ 5000. A

comparison with the color-curve of the objective shows that the only light which is effective in making the stellar images is included between λ 5200 and λ 5700. For, in the first place, the region between λ 4800 and λ 5200 is cut out by the plate itself; and again, the rays beyond λ 4800, toward the violet end of the spectrum, are so much out of focus that the light is enfeebled when it reaches the plate, because it is spread over a circle of considerable area. For example, light at λ 4500, to which the plate is sensitive, is 36 mm out of focus, and therefore reaches the plate as a circle about 2 mm broad in addition to the small diffraction disk. This spreading out prevents the blue rays from affecting the plate unless the exposure is much prolonged. It hardly seems necessary to remark that a screen which excludes blue rays is indispensable if the object to be photographed is a surface (such as the Moon, a nebula, or a very dense star cluster), for in this case the blue rays from different points in the source will overlap to such an extent as to make them as strong, at any one point of the photographic film, as though they were in focus at that point. On the other hand, the best screen for stellar work would not be one which cut out only the blue rays, but rather one which prevented the region λ 5200 to λ 5400 from reaching the plate. However, the improvement effected by such a screen as the latter would probably be slight and noticeable only under the best atmospheric conditions. For stellar-parallax measurements this possible improvement would not compensate for the presence of the screen. Consequently all the plates thus far secured for this work have been taken without interposing anything between the plate and the objective.

The following table shows the degree of accuracy with which the images can be bisected. The first star is the faintest that can be measured on a plate that has been exposed for five minutes under the best atmospheric conditions; the second is about the best size for bisection; and the third is nearly, but not quite, the largest image that can be bisected with confidence.

Magnitude	Diameter	Probable Error of one Setting
10.5	0.07 mm = 0″.75	0.0017 mm = 0″.018
9.2	0.20 2.1	.0012 .013
8.0	0.38 4.0	.0018 .019

It is interesting to compare these with the corresponding figures for the *Astrographic Catalogue* plates. The smallest images on these are perhaps half the size of the smallest on our plates; but the *angular* diameter of their images is nearly three times as large, and the probable error in arc of a single setting is three or four times greater than on the forty-inch plates.

The simple error of setting upon an image is well understood to form only a part of the total error in a photographic star place. A real test of accuracy is the agreement between different plates. Below are given four sets of residuals, which are fair samples of the accordance that we usually get. The initials refer to the two observers, Miss Louise Ware (L. W.) and the writer (F. S.). The brackets on the left indicate that the exposures inclosed by them are on the same plate. These residuals are derived from measurements of the double star Σ 2398; they are cleared of the effects of parallax and proper motion as well as refraction, aberration, etc.

From all these residuals we compute the probable error of one exposure to be $\pm 0\overset{''}{.}030$.

This agreement among different plates must be considered very satisfactory; it shows that only a few exposures will be necessary to reduce the accidental error in the computed parallax to a negligible quantity. There still remains, however, the more serious question of systematic errors. Previous experience in photographic as well as other work has shown that such errors are especially liable to creep into results, if the observations are made at large hour-angles. Consequently the present series of plates has been restricted to within two hours of the meridian. This can be done only at a sacrifice of "parallax factors;" that is, many of the plates (especially during the summer) are exposed when the effect of parallax upon the star's position is far from the maximum. This sacrifice is made possible by the minuteness of the accidental errors, which compensates in a large degree for the small coefficients. Another precaution which has been taken is to use the telescope only on one side of its pier. This entails an additional diminution of the parallax factors, but it is worth while; for it eliminates a possible distortion due to the objective, as the latter is always presented to the sky in the same position. Reversing the telescope is equivalent to rotating the objective 180°

RESIDUALS FOR THE DOUBLE STAR Σ 2398.

Brighter Star (8.5 Mag.)		Fainter Star (9.3 Mag.)	
F. S.	L. W.	F. S.	L. W.
−0″.080	−0″.053	−0″.059	−0″.035
− .021	+ .011	− .040	− .045
− .056	− .011	+ .013	− .005
− .059	− .061	− .011	+ .056
+ .069	+ .008	+ .053	+ .059
+ .013	+ .021	+ .035	+ .069
+ .037	+ .016	+ .037	+ .040
+ .043	+ .077	+ .008	+ .053
+ .021	− .013	+ .013	− .003
+ .032	+ .024	+ .024	+ .019
+ .013	− .027	− .035	− .045
− .027	− .061	− .035	− .043
+ .021	+ .045	+ .008	+ .027
+ .027	+ .013	+ .011	+ .053
− .045	− .077	+ .029	+ .048
+ .027	− .016	+ .048	+ .029
− .019	− .043	− .037	+ .029
− .059	− .051	− .016	+ .019
+ .056	− .005	− .008	− .043
+ .056	+ .080	+ .021	+ .037
− .051	+ .053	− .061	.000
+ .051	+ .008	+ .019	+ .035
+ .029	+ .128	+ .037	+ .053
+ .072	− .016	+ .003	− .003
− .035	− .093	− .003	− .064
− .053	− .045	− .027	− .011
− .003	− .043	+ .072	+ .024
− .048	− .059	+ .011	− .008
+ .053	− .016	+ .043	+ .051

around its optical axis. The great weight and diameter of the lenses, and the consequent sag when the telescope is pointed near the zenith, make this precaution especially important with the forty-inch telescope. The phenomenon known as "atmospheric dispersion," to which Dr. Rambaut has called attention,[1] can have no great effect on our plates; not only on account of the small hour-angles at which

[1] *Monthly Notices of the R. A. S.*, **55**, 123, 1895.

the exposures are made, but also because of the limited region of the spectrum (λ 5200 to λ 5700) which is here concerned in forming the image, whatever the color of the star may be.

These precautions, and some less important ones that need not be described here, seem to have been successful in keeping out systematic errors, so far as may be judged from the data thus far accumulated. Some examples of the parallaxes computed from the plates are given below, but it must be borne in mind that these results are only preliminary. Definitive corrections and additional plates will no doubt modify them to some extent.

Krueger 60 (R. A. = 22^h 24^m, Decl. = $57°$ $10'$).

This star was put upon the list at the suggestion of Professor Barnard, who inferred (correctly it appears) that the star has a large parallax.[1] One of the stars of *Krueger* 60 is itself a binary in comparatively rapid orbital motion, the components now being about $3''$ apart and of magnitudes 9.1 and 11.0 respectively; the system has an annual proper motion of nearly a second of arc. Our plates furnish the following values of the parallax for the brighter component; these are independent of each other, except that Miss Ware's measurements and the writer's refer to the same plates, eight in number, containing twenty exposures.

Parallax	Weight	
+0.268	3	by L. W. from R. A., 5 distant comparison stars.
+0.277	3	by F. S. from R. A., 5 distant comparison stars.
+0.226	1	by L. W. from R. A., 1 close comparison star.
+0.258	1	by F. S. from R. A., 1 close comparison star.
+0.292	1	by L. W. from Decl., 5 comparison stars.
+0.301	1	by F. S. from Decl., 5 comparison stars.

Means, R. A.	+0″.265,	weight 8
Decl.	+0.296,	weight 2
Means, L. W.	+0.264,	weight 5
F. S.	+0.278,	weight 5
Means, 5 comparison stars	+0.278,	weight 8
1 comparison star	+0.242,	weight 2

This is the first determination of the parallax of this star; if it

[1] *Astronomical Journal*, **23**, 169, 1903.

should be confirmed by other measurements, this faint star would appear to be one of the Sun's nearest neighbors.

Fedorenko 1457, 8 (R. A. = $9^h\ 7^m$, Decl. = $53°\ 7'$).

This is a wide double, each component being of about the eighth magnitude. Values of the parallax of both stars were deduced from only four plates containing eleven exposures. These are not sufficient to determine the proper motion, which is, however, known well enough for present purposes. In this case we have the equation $\Delta\pi = 0.21\ \Delta\mu$, which shows the dependence of the computed parallax upon the assumed proper motion; that is, any error in the latter will affect the parallax by about one fifth this error.

Parallax		
+0.231	Preceding star, measures by F. S.	
+0.231	Preceding star, measures by L. W.	
+0.205	Following star, measures by F. S.	
+0.226	Following star, measures by L. W.	
Means,	F. S.	+0.″218
	L. W.	+0.228
Means,	Preceding star	+0.231
	Following star	+0.216

Peter obtained +0.″18 for the parallax of this star by means of the heliometer.

Struve 2398 (R. A. = $18^h\ 41^m$, Decl. = $59°\ 29'$).

This double now has a separation of about 17″. The components are of magnitude 8.5 and 9.3 respectively, and both could be measured upon our plates. The system is in rapid motion as a whole, 2.″3 per annum. A comparison between the photographic measures and earlier micrometer work upon these stars shows that there is considerable orbital motion, the relative directions having changed nearly 90° since Struve's observations. This motion is somewhat surprising in view of the wide separation and the faintness of the components, and makes the system nearly unique. The values of the parallax given below rest upon eleven plates containing twenty-nine exposures. As in the previous case, the plates are not yet sufficient to determine the proper motion, which has to be assumed for the present. The orbital motion was taken into account in this connection. We have for both stars $\Delta\pi = 0.40\,\Delta\mu$.

Parallax	
+0.287	by F. S. from the R. A., for the brighter star.
+0.299	by L. W. from the R. A., for the brighter star.
+0.283	by F. S. from the R. A., for the companion.
+0.292	by L. W. from the R. A., for the companion.

Means, Bright star	+0."293
Companion	+0.288
Means, L. W.	+0.296
F. S.	+0.285

Flint obtained +0."32 for this parallax, using Kapteyn's meridian-circle method; and Lamp, by measuring differences of declination with a micrometer, made it 0."35.

YERKES OBSERVATORY,
August 5, 1904.

ON THE TRANSITION FROM PRIMARY TO SECONDARY SPECTRA.

By P. G. NUTTING.

THE work here described was undertaken to determine as definitely as possible the conditions under which secondary spectra are produced, and to study the effect of slightly varying these conditions in the critical state when both spectra are present. Plücker and Hittorf[1] in 1865 proved that several of the elementary gases may emit two entirely different spectra. With a large capacity in parallel with the tube of conducting gas, they obtained a disruptive discharge which emitted a bright line spectrum, called by them the secondary spectrum. I have undertaken to determine how much capacity was necessary to just produce the secondary spectrum in different gases, and how this critical capacity varies with the wave-length, with the density of the gas, the amount of inductance and resistance in circuit, distance apart of electrodes, and sectional area of the discharge.

Photographic methods were employed throughout. Spectra obtained under varied conditions were photographed side by side on the same plate, so that the minutest changes could be observed and followed. For this purpose a large model Fuess quartz spectrograph was used. This was provided with a large flint glass prism giving a spectrum about 15 cm long from 300 to 600 $\mu\mu$. Ten spectra could be recorded on the same plate. A large glass condenser was used, composed of twenty plates well separated and provided with mercury cups so that the capacity could be varied by a plate at a time. Current was supplied by transformers of 1,000, 2,000, and 5,000 volts, and by a set of generators giving 5,000 volts continuous current. For inductance, a Seibt tuning solenoid of 120 20 cm turns was used. The greater part of the work was done with ordinary short stout Plücker tubes made by Boehm, of Chicago. These had electrodes about 4 cm apart and capillary portions 2 mm in diameter and 12 mm

[1] *Phil. Trans.*, **155**, 1–29, 1865.

long. To keep the pressure of the inclosed gas more nearly constant, a half-liter bulb was sealed to each tube while in use.

With a tube of air at 13 mm pressure, spectra were photographed with capacities of 0.12, 0.09, 0.06, 0.03, and 0.0 microfarad in parallel. A sudden change from secondary to primary spectrum was found to occur at a capacity of 0.04 mf, a capacity equivalent to that of about fourteen one gallon Leyden jars. Adding capacity above 0.06 mf produced little if any effect, nor do secondary lines appear in the primary spectrum until the capacity is nearly 0.03 mf.

Critical capacity and wave-length.—Drawing a line separating primary and secondary spectra on the photographic plate (see Plate XI), the ordinates of the curve represent roughly critical capacity, the abscissas wave-length. The curve drops off very steeply toward short wave-lengths, and in spite of the greater dispersion, indicating that for waves perhaps not shorter than 300 $\mu\mu$ the critical capacity becomes infinite. Critical capacity expressed as a function of wave-length appears to be of the form

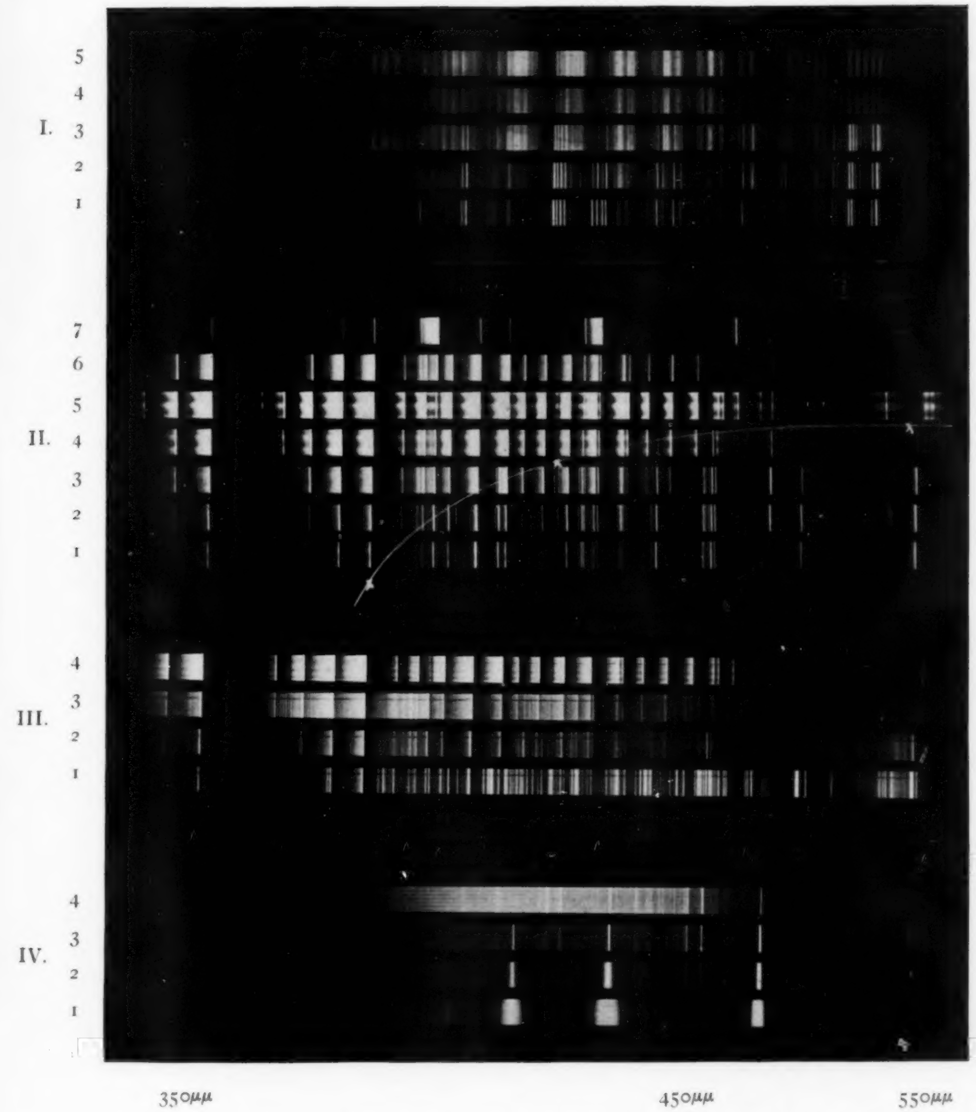
$$C = ae^{-b/(\lambda-\lambda_0)}$$

Approximate numerical results for air are given in the table:

Pressure	Wave-Length in $\mu\mu$		
	350	450	550
20 mm........	0.07	0.03	0.02 mf.
1 mm........	0.15	0.06	0.03 mf.

Critical capacity and pressure.—The critical capacity increases slightly as the pressure decreases down to about 1 mm air pressure, when it suddenly becomes infinite; *i. e.*, no amount of capacity will cause the secondary spectrum to appear unless an external spark-gap is used. This critical minimum pressure at which the disruptive discharge becomes possible is considerably lower for hydrogen (about 0.1 mm), but depends on the condition of the surface of the electrodes and the presence of impurities. At high pressures the critical capacity continues to gradually decrease. Using a special tube, the distance apart of whose electrodes could be varied to suit the potential (5,000 volts) employed, the critical capacity of air and hydrogen was observed up to atmospheric pressure, when it was found to be about one-fourth

PLATE XI.



I. Sulphur: 1, secondary; 5, primary; 2, 3, 4, combinations of primary and secondary showing their co-existence. II. Nitrogen at 1 mm pressure: 1, taken with 0.12 mf capacity; 2, with 0.09 mf; 3, with 0.06 mf; 4, with 0.03 mf; 5, primary (anode), no capacity; 6, same as 3, but taken in bulb instead of capillary; 7, *N* cathode glow. III. Inductance effect in nitrogen at 21 mm pressure: 1, 0.04 mf capacity, no inductance; 2, same capacity, 0.12 m. h. inductance; 3, 0.9 m. h. inductance; 4, *N* primary, neither capacity nor inductance. IV. Hydrogen under same conditions as the Nitrogen in III.

what it was at 10 mm pressure. This would make the critical capacity roughly proportional to the cube root of the pressure, or inversely proportional to the mean distance apart of the molecules.

Critical capacity and nature of the gas.—Hydrogen, sulphur, nitrogen, oxygen, bromine, and iodine were tested and the critical capacity found to be practically the same for all for the same pressure and wave-length. Critical capacity is more sharply marked in sulphur, nitrogen, and iodine. With hydrogen, the lines of the secondary (four-line) spectrum invariably appear in the primary (many-line) spectrum, the capacity of the wires leading to the tube being a considerable factor in this dominance. All the substances show the same great increase in critical capacity for the very short wave-lengths and decrease with increasing pressure..

Critical capacity appears to be nearly or quite independent of the voltage employed (up to 5,000 volts) and of the separation of the electrodes. Tests were made with the electrodes but 3 mm apart. At much shorter distances the metallic lines from the electrodes become prominent at higher pressures.

In the capillary of a Plücker tube the critical capacity is less than in the bulb, or less than in a tube without a central constriction. This is confirmatory to a view expressed in a previous paper,[1] that the production of a secondary spectrum was not so much the effect of capacity *per se* as of increased current density.

Critical capacity and inductance.—The effect of introducing inductance is always to relatively weaken the secondary and enhance the primary spectrum. But putting in a certain inductance is by no means equivalent in its effect to taking out a definite corresponding capacity. Inductance was added in steps of 0.008 millihenry. The first inductance added, though very small, weakened the secondary spectrum very markedly and introduced primary lines, and this whether the capacity used was just above the critical capacity or five times that amount. Adding more and more inductance produces less and less additional effect. Apparently no amount of inductance, however great, will *completely* annul the effect of any capacity, however small. Capacity and inductance effects are shown graphically in the figure. Ordinates represent the change from primary to secondary spectra.

[1] ASTROPHYSICAL JOURNAL, May 1904.

Critical capacity and resistance.—The effect of resistance is as pronounced as that of inductance in changing the secondary spectrum back to the primary. Even as little as 20 ohms (non-inductive) resistance brings in primary lines, while 100 ohms gives a nearly pure primary. The resistance effect curve has very nearly the same form as the inductance effect curve, as shown in the figure.

Critical capacity of mixtures.—Mixtures of hydrogen and nitrogen, sulphur and hydrogen, iodine and nitrogen, iodine and hydrogen, nitrogen and sulphur, hydrogen and oxygen, and mercury and nitrogen were tested, and each component was found to have its own

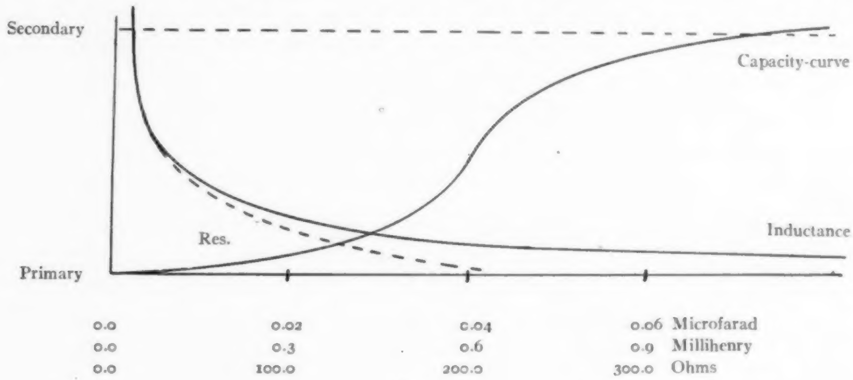


FIG. 1.

critical capacity independently of the presence of the other. In a previous paper[1] it was shown that the relative intensities of two *primary* spectra (of a mixture of gases) depend on the relative atomic weight of the component gases, while the relative intensities of the *secondary* spectra depend only on the relative numbers of the different kinds of atoms present. For example, it is easy to prepare a tube of mixed hydrogen and nitrogen that shows a ("four-line" secondary) hydrogen spectrum with capacity, but a nitrogen spectrum without. To detect spectroscopically, small amounts of a gas of low atomic weight mixed with a heavier, secondary spectra are most favorable while it is better to work with primary spectra when seeking evidence of a gas of large atomic weight. If an unknown gas exhibit two widely different spectra under different excitation, that is rather evidence that it consists of but a single gas than that it is a

[1] *Ibid.*, March 1904.

mixture of two different gases. When working with gases of large atomic weight, impurities of low atomic weight will be of little consequence, particularly if the primary spectrum is used.

To obtain a pure primary spectrum, then, it is necessary to avoid introducing capacity of any kind (particularly long lead wires) into the tube circuit. It is useless to attempt to neutralize the effect of even a small capacity by introducing inductance or resistance. To obtain a pure secondary spectrum, inductance and resistance are to be avoided, and at least 0.05 microfarads capacity must be added. With more than 10 ohms resistance or 0.01 millihenry inductance in the tube circuit, it is useless to attempt to get a pure secondary spectrum by simply adding an excess of capacity.

In a previous paper[1] the hypothesis was advanced that whole atoms radiate primary spectra, while atoms from which one or more electrons have been torn radiate secondary spectra. The atoms of the metallic elements appear to be so easily ionized that the feeblest current that will excite luminosity is more than sufficient to disrupt the atom, hence only electro-negative elements give both primary and secondary spectra. Large capacity would then produce always secondary spectra by increasing the intensity and suddenness of the current-wave through a gas, since a wave of large amplitude and steep wave-front would be vastly more effective in tearing apart an atom. Under these conditions we should expect inductance to tend to reduce secondary to primary spectra, since it reduces the slope of the current-wave; resistance would produce the same effect by lowering the wave-amplitude. Increasing the cross-section of the discharge would have the same effect as increasing the resistance. Critical capacity would be greater at the more refrangible end of the spectrum because modes of vibration of higher frequency would require a steeper current-wave to affect them; or perhaps it is equivalent to say that the smaller orbits are most stable. Critical capacity being a function of spectral wave-length, I consider as strong evidence that each spectral line is due to a different electron and *not* merely to one of many modes of vibration possessed by (say) a ring of electrons.

NATIONAL BUREAU OF STANDARDS,
Washington, D. C.,
June 1904.

[1] *Ibid.*, p. 243, May 1904.

FAINT STARS NEAR THE TRAPEZIUM IN THE *ORION* NEBULA.

By J. A. Parkhurst.

The lists of new variable stars in the Great Nebula of *Orion*, lately published by Wolf[1] and Pickering,[2] include none near the trapezium, as the faint stars are hidden by the brightness of the nebula on plates taken with short-focus instruments. Hence an examination has been made of the nine negatives taken by Mr. Ritchey with the Yerkes forty-inch refractor in 1900 and 1901. As these negatives were taken at the visual focus with a yellow screen and isochromatic plates, the photographic magnitudes will correspond quite closely with the visual values. The examination has been limited to the region within 2′ of arc of the trapezium star θ', and only those stars have been included in the list which appear on several of the plates, thus giving some basis for an opinion as to their variability. The exposure times of the plates range from one to four hours, but the stars are usually best shown with one-hour exposure.

NOTATION AND POSITIONS.

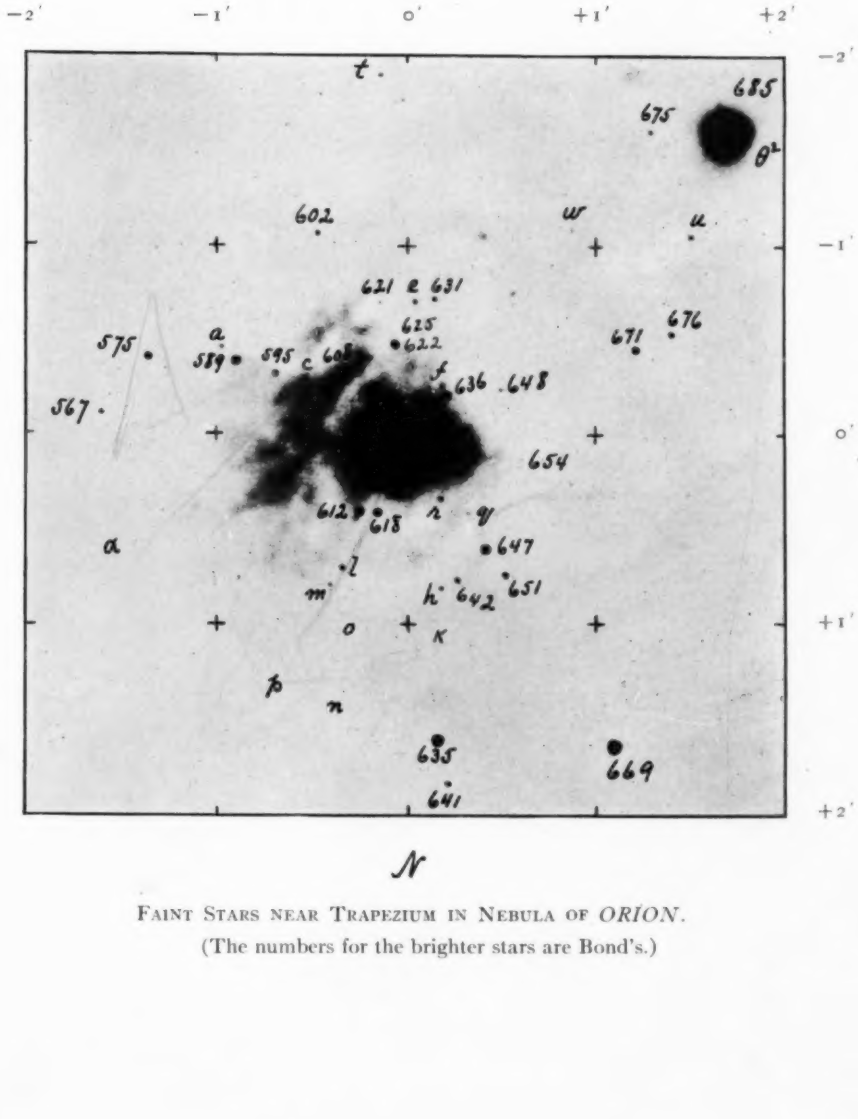
Plate XII is an enlargement from the negative of 1900 October 17, $15^h\ 10^m$ to $16^h\ 10^m$, Central Standard Time. The orientation and scale were deduced from the positions of the stars Bond 558, 657, and 708, as measured by Scheiner[3] and reduced to the epoch of Bond's catalogue, 1857. Bond's numbers are used for all stars given by him, and letters are assigned by the writer to the faint stars not in Bond's list. As a check on the positions, a similar enlargement was made from the negative of 1901 Dec. 28, and the co-ordinates of the stars relative to θ^1 were measured with a millimeter scale on the two enlargements. The mean differences in the resulting positions were $0\overset{''}{.}7$ in R. A. and $0\overset{''}{.}5$ in Dec. (differences greater than $1''$ occurring only in case of faint and poorly defined stars), thus insuring correct identifications.

[1] *Astronomische Nachrichten*, **164**, 393, 1904.

[2] *Harvard Circulars*, **78**, **79**.

[3] *Potsdam Publications*, **11**, 63, 64.

PLATE XII.



Faint Stars near Trapezium in Nebula of *ORION*.

(The numbers for the brighter stars are Bond's.)

Omitting the six trapezium stars and θ^2, a comparison of the number of stars shown or measured in this region by different observers follows:

TABLE I.

Bond	*Harvard Annals*, **5**	visual 23
W. H. Pickering	*Harvard Annals*, **32**	photographical 10
E. C. Pickering	*Harvard Circulars*, **78**, **79**	photographical 0
Scheiner	*Potsdam Publications*, 11	photographical 0
Yerkes 40-inch	....	photographical 42

For convenience of comparison, the magnitude estimates are based on Bond's scale, though for the faint stars his numbers are probably too large, perhaps by 0.5 at fifteenth magnitude. These estimates are evidently subject to greater uncertainty than in case of stars in a region free from nebulosity, as the background varies much in density on plates of different exposure times. It should also be stated that Plate XII does not preserve the relative magnitudes of the outlying stars, as the reproduction was made to include as little nebulosity as possible.

Table II gives the results of the investigation, also for comparison Bond's co-ordinates and magnitudes, the headings of the columns being sufficient explanation of the contents.

VARIABLE STARS.

Bond or Struve has suspected of variability all the stars in this region visible to them except 625 and 635. The present investigation seems to prove the variation of 642 with a range of at least 2.5 magnitudes, and of 654 with a range of 4 magnitudes. Less certainly proved is the variability of 567, 589, 608, 625, 641, 675 and *s*, which with a few other stars require special mention.

Bond 601. Not seen except as a condensation in the nebula; only glimpsed by Bond on one night; perhaps is not a star.

Bond 608. In dense nebulosity; comparisons uncertain.

Bond 621 and 625. Only glimpsed by Bond on one night and positions estimated by him, not measured.

Bond 642. It is perhaps a question whether Bond saw this star or my *h*. At maximum the variable is brighter than *h*, but at minimum it is about 2 magnitudes fainter.

TABLE II.

Designation	Yerkes					Bond		
	Co-or. from θ^1		Magnitude			Co-or. from θ^1		Magnitude
	R. A.	Dec.	Mean	Brightest	Faintest	R. A.	Dec.	
567	−1′ 37″	−0′ 6″	13.4	13.2	13.6	−1′ 43″	−0′ 8″	13.9
a	−1 27	+0 36	15.8	....	....	......	......	....
575	−1 22	−0 23	11.8	....	....	−1 25	−0 22	11.9
a	−0 59	−0 27	15.0	....	....	......	......	....
589	−0 55	−0 23	var?	12.4	13.3	−0 57	−0 20	12.7
595	−0 42	−0 18	15.0	....	....	−0 47	−0 15	13.9
p	−0 37	+1 15	14.8	....	....	......	......	....
c	−0 32	−0 16	15.6	....	....	......	......	....
d	−0′27	−0 11	15.8	....	....	......	......	....
602	−0 28	−1 3	14.2	....	....	−0 33	−1 8	14.3
m	−0 24	+0 49	15.8	....	....	......	......	....
l	−0 20	+0 43	14.0	....	....	......	......	....
n	−0 20	+1 22	14.8	....	....	......	......	....
608	−0 19	−0 18	var?	14.4	15.2	−0 24	−0 18	14.3
o	−0 16	+0 58	15.8	....	....	......	......	....
612	−0 15	+0 25	13.3	....	....	−0 16	+0 25	13.5
618	−0 9	+0 26	13.3	....	....	−0 10	+0 25	13.1
621	−0 9	−0 41	15.8	....	....	−0 8	−0 36	15.6
t	−0 9	−1 53	14.2	....	....	......	......	....
622	−0 4	−0 27	12.7	12.0	12.9	−0 8	−0 28	12.7
625	−0 4	−0 31	var?	15.2	16.5	−0 4	−0 28	15.[illegible]
e	+0 2	−0 41	14.7	....	....	......	......	....
s	+0 7	0 0	15.6	....	....	......	......	....
631	+0 9	−0 42	14.1	....	....	+0 3	−0 42	14.3
k	+0 9	+1 0	15.6	....	....	......	......	....
635	+0 10	+1 37	9	....	....	+0 8	+1 38	10.5
h	+0 10	+0 50	14.7	....	....	......	......	....
r	+0 11	+0 21	16.0	....	....	......	......	....
j	+0 11	−0 14	16.1	....	....	......	......	....
641	+0 13	+1 51	13.1	12.6	13.3	+0 12	+1 51	14.8
636	+0 13	−0 12	13.8	....	....	+0 8	−0 9	13.3
642	+0 16	+0 47	var.	14.0	16.6	+0 13	+0 48	15.6
q	+0 19	+0 30	16.8	....	....	......	......	....
647	+0 25	+0 37	11.2	....	....	+0 23	+0 38	12.1
648	+0 30	−0 13	14.4	13.6	14.7	+0 24	−0 9	14.3
651	+0 31	+0 45	12.6	....	....	+0 29	+0 48	13.1
654	+0 36	+0 8	var.	11.9	16.1	+0 33	+0 10	12.3
w	+0 52	−1 3	15.9	....	....	......	......	....
671	+1 12	−0 26	11.7	....	....	+1 10	−0 24	11.5
675	+1 17	−1 35	14.6	14.4	14.8	+1 15	−1 33	15.2
676	+1 24	−0 31	12.7	....	....	+1 19	−0 28	13.1
u	+1 30	−1 2	14.5	....	....	......	......	....

s. This is a double star discovered by Barnard with the Lick 36-inch in 1888. It is called "excessively faint" by Burnham,[1] the components being rated as 16.0 and 16.5 magnitude. It is well shown on the negatives of one-hour exposure, and, if of ordinary color, seems to be at least a magnitude brighter than in 1888. It is too near the trapezium to be shown on the print.

Bond 567. Rated by Holden as 16.3 magnitude, 1878 Jan. 5, with Washington 26-inch.[2]

Bond 641. Rated by Holden as 16 magnitude, 1874 Jan. 6.[3]

YERKES OBSERVATORY,
July 28, 1904.

[1] *Publications of the Lick Observatory*, **2**, 48.

[2] *Washington Observations*, 1878, *Appendix I*, 182.

[3] *Ibid.*, page 181.

ON SOME RESULTS OBTAINED BY THE D. O. MILLS EXPEDITION TO THE SOUTHERN HEMISPHERE.[1]

By W. H. WRIGHT.

THE Observatory of the D. O. Mills Expedition to the Southern Hemisphere was installed on the summit of Cerro San Cristobal in the city of Santiago de Chile during the southern winter of 1903, and observing was commenced on September 11. In advance of a detailed description of the observatory, instruments, and methods of work, which will be forthcoming, it is desirable to publish certain results secured during the progress of the work. It will be sufficient for present purposes to state that the equipment of the expedition includes a reflecting telescope of 94 cm clear aperture, Cassegrainian mounting, and a powerful three-prism spectrograph. With this combination, results are being secured comparable in point of accuracy with those obtained with the Mills spectrograph at Mount Hamilton. Three hundred and eight successful spectrograms were obtained up to June 1, 1904.

The following stars have been found to have variable radial velocities:

β Doradus ($\alpha = 5^h\ 32^m.7$; $\delta = -62°\ 33'$).

Date	Velocity	Measured by
1903, Sept. 20	+ 1.4 km	Palmer
Dec. 21	+16.1	"
1904, Jan. 12	+28.0[2]	"
Jan. 22	+28.5	"

w Velorum ($\alpha = 8^h\ 56^m.3$; $\delta = -40°\ 52'$).

1904, Jan. 21	+ 3.0[2] km	Palmer
Mar. 8	+13.7	"
Mar. 28	+ 7.5	"

[1] Also to appear as a *Bulletin* of the Lick Observatory.

[2] Rough measurements of very poor plates.

l Carinae ($\alpha = 9^h\ 42^m.5$; $\delta = -63° 3'$).

This is a variable star having a period, according to A. W. Roberts,[1] of 35.523 days. The light-variation, according to the same authority, is irregular. Our observations are as follows:

Date	Velocity	Measured by
1904, Apr. 18	+10 km	Wright
Apr. 30	+22	"
May 8	−15	"

κ Pavonis ($\alpha = 18^h\ 46^m.6$; $\delta = -67° 21'$).

This is also a variable star, having, according to the same authority, a regular light-curve, and a period of 9.091 days.

Date	Velocity	Measured by
1904, May 12	+40.1 km	Palmer
June 6	+28.9	"
June 22	+26.5	"

τ Sagittarii ($\alpha = 19^h\ 0^m.7$; $\delta = -27° 49'$).

1902, Aug. 17	+34.0 km	*
1904, May 12	+51.0	Palmer
June 7	+59.7	"

The variations in the velocities of *β Doradus* and *w Velorum* were detected by Dr. Palmer.

Observations of the radial velocities of the components of *α Centauri* have been secured, as follows:

α_1 Centauri (fainter component).

Date	Velocity	Measured by
1904, Feb. 25	−18.90 km	Wright
Mar. 4	−18.69	"
June 23	−19.70	"

[1] *Astronomical Journal*, **21**, 81, 1901.

* Observation made at Mount Hamilton.

α_2 Centauri.

1904, Feb. 21.................	−24.02 km	Wright
Mar. 4.................	−24.20	"
June 23.................	−24.58	"

In addition to these observations, the spectra were photographed on May 29, using a short camera designed for work on fainter stars. The negatives are rather poor, the spectra being over-exposed in the region of good definition. However, they were measured soon after being taken. The results are:

α_1 Centauri	*α_2 Centauri*
1904, May 29 −19.70 km	May 29 −24.30 km

For reasons stated, it was considered that these observations would not add to the value of the final determination. They have accordingly not been used.

A difference in the radial velocities of the components is shown to exist.

In order to make the observations as nearly as possible differential, exposures after that of February 25 were made on the two components in quick succession, the adjustments and the settings of the instrument remaining unchanged. The differences taken by pairs are:

Date	$V_1 - V_2$
1904, February 21–25 - - - - -	5.12 km
March 4 - - - - -	5.51
June 23 - - - - - -	4.88
Mean - - - - - -	5.17 km

As to the cause of the difference, two assumptions may in general be made in a case of this sort.

1. The difference may be due to the relative orbital motion of the two components.

2. It may be due, in part at least, to one of the components being a spectroscopic binary.

Under the first of these hypotheses the parallax of *α Centauri* may be computed, as the visual orbit of the pair is accurately known. Assuming the elements determined by Roberts,[1]

[1] *A. N.*, **133**, 105, 1893; also **139**, 7, 1895.

$$\begin{aligned} T &= 1875.715 \\ P &= 81.185 \text{ years} \\ e &= 0.52865 \\ \lambda &= 52^\circ\ 0'\ 58'' \\ i &= 79^\circ\ 21'\ 36'' \\ \Omega &= 25^\circ\ 5'\ 50'' \\ a &= 17''.71 \end{aligned}$$

the computation is readily effected by means of the following formulæ adapted from the work of Lehmann-Filhés:

$$n = \frac{2\pi}{86400 \times 365.26 \times P} ,$$

$$a = \frac{\Delta V \sqrt{1-e^2}}{n \sin i\, [e \cos \lambda + e \cos (V+\lambda)]} ,$$

$$\pi'' = \frac{a''}{a} R ,$$

$$m_1 + m_2 = \frac{a^3}{R^3 P^2} ;$$

where

R = mean heliocentric distance of the Earth in kilometers,
a = mean distance of the components of *α Centauri* in kilometers.
n = mean angular motion of *α Centauri* in circular measure, per second of time.
ΔV = observed difference in radial velocity of the two components.
m_1 and m_2 = masses of α_1 and α_2 *Centauri* in terms of the mass of the Sun.

Dr. Palmer has at my request performed the computations. His results are

$$\pi = 0''.76 \text{ ,[1]}$$
$$a = 3.46 \times 10^9 ,$$
$$m_1 + m_2 = 1.9 .$$

The computed probable errors for these quantities, depending on the residuals from the mean of the three determinations of differential velocity are, for π, $\pm 0''.03$, and for $m_1 + m_2$, ± 0.2. Being based

[1] Gill and Elkin's value of the parallax of *α Centauri*, from heliometer observations is

$$\pi = 0''.75 \pm 0''.01 ,$$

relative to the comparison stars used, of average magnitude 7.6.

on such a small number of observations, the weights are quite uncertain. Furthermore, they do not take into consideration the uncertainties in the elements of the orbit, nor other sources of error to be discussed later. In this connection it should be remembered that the orbit of *α Centauri* is among the most accurately known of double star orbits.

The second hypothesis, while not a probable one, cannot in general be neglected in a discussion of this nature. Radial velocity determinations on an extensive scale have developed the fact that at least one star in every seven has a variation in velocity great enough to be detected by the powerful spectrographs now in use. If among m stars there are n spectroscopic binaries, then the chance that of any two taken at random at least one should belong to this class is

$$\frac{n(2m-n-1)}{m(m-1)}$$

which, if m be large, reduces approximately to

$$\frac{n}{m}\left(2-\frac{n}{m}\right),$$

Assuming $\frac{n}{m}=\frac{1}{7}$, it appears that the probability of at least one of a pair of stars being a spectroscopic binary is a little greater than one-fourth. The probability in the case under discussion is somewhat lessened by the fact that the general ratio of one to seven is influenced to a great extent by short-period variation, which is not shown to exist by these observations extending over four months. At the same time, it must be confessed that we are working very much in the dark, as the ratio may be different in the cases of telescopic double stars from what it is for stars apparently single. But be the probability great or small, the actual existence of variable velocities in the components of *ζ Ursae Majoris, α Geminorum, κ Pegasi,* and other well-known double stars warns us to accept with some reservation parallax determinations based on observations of radial velocity extending over only a short period of time.

It seems advisable to publish these observations now, as it is likely to require a great many years to determine whether the observed differences in radial velocity vary according to the stars' orbital motion.

It is to be noted that considerations similar to the foregoing apply with equal force to the older and more direct method of determining parallax, as the dimensions of the orbits of many spectroscopic binaries are of the order of magnitude of those of the Earth's orbit. In determining by either method it is therefore desirable that the observations should extend over a considerable period of time.

In conclusion, I may be permitted to recapitulate the advantages of the spectroscopic method (where applicable) over the direct method.

1. No assumption is made as to the great distance of certain comparison stars.

2. The accuracy of the determination of the star's distance is to a certain degree independent of this distance.

3. The quantities upon which the parallax depends will usually be of an order lower than that of the probable error of measurement.

The brighter component of α *Centauri* has a solar-type spectrum. In the spectrum of the fainter component the heavy iron lines are much more pronounced, and the calcium absorption is exceedingly heavy. According to Roberts,[1] the masses of the stars are nearly equal, the exact ratio being $\frac{51}{49}$ in favor of the brighter component. When the relative velocity in the line of sight shall have changed, this ratio can be determined from velocity observations, though probably not with such accuracy as from heliometer measurements from neighboring stars.

OBSERVATORY OF THE D. O. MILLS EXPEDITION
TO THE SOUTHERN HEMISPHERE,
Santiago de Chile, June 28, 1904.

[1] *A. N.*, **139**, 10, 1895.

MINOR CONTRIBUTIONS AND NOTES.

A LIST OF FIVE STARS HAVING VARIABLE RADIAL VELOCITIES.[1]

IN the course of the line-of-sight work with the Lowell spectrograph, the following five stars have been discovered to be spectroscopic binaries. These are additional to those previously announced. Since some of the plates employed in these determinations have been incompletely measured and reduced, the time is given only to the day. The letters S and L before the plate numbers refer respectively to the short and to the long camera.

α Andromedæ ($a = 0^h\ 3^m.2$; $\delta = +28°\ 33'$; Mag. $= 2.1$).

The following observations of the radial velocity of this bright star show it to be a spectroscopic binary.

Plate	Date	Velocity
S 470	1902 Oct. 30	−40 km
L 1248	1903 Nov. 25	−42
L 1253	Nov. 26	−40
L 1290	Dec. 1	−34
S 1306	Dec. 14	−27
S 1318	Dec. 19	−24
S 1328	1904 Feb. 10	+16
S 1331	Feb. 11	+20
S 1338	Feb. 17	− 5
S 1341	Feb. 29	−37
S 1347	Mar. 4	−45
S 1349	Mar. 6	−44
S 1402	May 22	+10

These velocities depend principally upon displacements of the hydrogen line $H\gamma$ and the magnesium line $\lambda\,4481$. The helium line at $\lambda\,4472$ is measurable on a few of the plates. Owing to the character of the spectrum and the poor quality of some of the plates, these values for the velocity may be in error a few kilometers.

This star was observed by Vogel and Scheiner in 1889.93 to have a velocity of +4.5 km, and when plate L 1248 was found to give a velocity differing by 45 km, the binary character of the star seemed certain. (Plate S 470 had not been previously measured owing to a badly overexposed

[1] *Lowell Observatory Bulletin* No. 11.

comparison spectrum.) These observations seem to indicate a period of about one hundred days, and a highly eccentric orbit.

α *Librae* ($a=14^h\ 45^m.4$; $\delta=-15°\ 37'$; Mag. $=2.3$).

The observations of this bright star showing the variation in its radial velocity are the following:

Plate	Date	Velocity
S 1406	1904 May 24	−60 km
S 1460	June 21	−20
S 1482	June 27	+ 4
S 1500	July 6	+20

The spectrum of this star is somewhat more advanced than that of *Sirius*, and is quite similar to that of α *Piscis Austrini*. There are numerous metallic lines, but they are poorly defined and not suitable for accurate measurement. The appearance and behavior of the hydrogen line $H\gamma$ suggest that both components are bright.

σ *Scorpii* ($\alpha=16^h\ 15^m.1$; $\delta=-25°\ 21'$; Mag. $=3.0$).

The variation in the radial velocity of this star was discovered from the second plate. The observations are as follows:

Plate	Date	Velocity
S 1451	1904 June 18	−25 km
S 1475	June 25	+25
S 1481	June 26	+17
S 1506	July 7	− 5

The spectrum is of the *Orion* type, and the lines are quite well defined.

X Sagittarii ($\alpha=17^h\ 41^m.3$; $\delta=-27°\ 48'$; Mag. $=4.9$).

This is the visual variable *X Sagittarii*, having a period of seven days. Only two spectrograms of the star have thus far been secured. They give the following values for the radial velocity.:

Plate	Date	Velocity
S 1455	1904 June 19	+ 1 km
S 1464	June 22	−22

This range is not great, but the spectrum contains many well defined lines, and there is no reason for doubting the reality of the variation in the

velocity. The spectrum is intermediate between that of *α Canis Minoris* and that of the Sun.

ε Capricorni ($\alpha = 21^h\ 31^m.5$; $\delta = -19° 54'$; Mag. $= 4.5$).

The variable velocity of this star was discovered in October 1903 from the fourth plate. The observations thus far obtained are the following:

Plate	Date	Velocity
S 1004	1903 Aug. 21	−40 km
S 1015	Aug. 24	−42
S 1046	Sept. 7	−45
S 1166	Oct. 28	−16
S 1170	Nov. 2	−15
L 1233	Nov. 22	−27
S 1469	1904 June 23	−23
S 1499	July 5	+ 1
S 1504	July 6	+ 6

The spectrum of this star is of the *Orion* type and is peculiar. The hydrogen line $H\gamma$ is, in general, very sharply defined, and the determinations depend principally upon the measures of this line alone. On some plates other ill-defined lines appear, and it may be that both components are bright.

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JULY 8, 1904.